University Ranking Publications: 
To Manipulate or Not to Manipulate*

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Abstract
We develop a multi-period theoretical model to characterize the relationship between a publication that ranks universities and prospective attendees – high school students – who might view the ranking and use it to help decide which university to attend. We assert that published rankings not only offer information about the objective quality of universities, but also have an effect on the prestige of universities, which is an element in students’ utility functions beyond objective quality elements. We show that a prestige effect can incent publications to take actions that are not in the best interest of the students; an example would be the excessive changes to ranking methodology that U.S. News & World Report (USNWR) is usually accused of. We show that if a ranking that uses an attribute-and-aggregate ranking methodology (the ranking methodology publications like USNWR and BusinessWeek use) creates prestige, then the publication (a) optimally chooses attribute score weights that do not match student preferences and (b) changes these attribute score weights over time even if there are no changes in student preferences. If a prestige effect is not present, then, according to our model, the publication optimally chooses attribute score weights that match student preferences. We use our model to characterize a socially-optimal ranking methodology – one that maximizes the sum of the publisher’s profit, the utilities of students who view the ranking, and the utilities of the students who do not view the ranking – and show that the socially-optimal ranking methodology evolves over time toward a stable ranking that diverges from the publisher’s optimal ranking. We conclude by discussing how students should deal with published rankings in the current environment, and what types of ranking methodologies might be developed to better represent student preferences.
1 Introduction

A number of publications rank universities, including *U.S. News & World Report* which ranks U.S. colleges and universities (hereafter *USNWR*), *BusinessWeek* which ranks U.S. undergraduate business and MBA programs, *The Times* which ranks international universities, and *Maclean's* which ranks Canadian universities, among many others. Presumably, these publications rank universities for business reasons – the rankings generate revenue directly through selling access to detailed information about universities or indirectly by selling advertising on their ranking websites, the expected returns for which are determined partially by the amount of traffic the website generates.

College and university rankings receive frequent criticism about the methodologies they employ, most of which are of the attribute-and-aggregate form. In fact, the sentiment about college rankings in general and *USNWR* specifically is so significant that a Wikipedia page is devoted solely to the criticism of college rankings. One common criticism about the *USNWR* ranking is that the publication strategically changes either the method it uses to measure attributes or the attribute score weights too frequently to represent either changes in the ways universities deliver education or changes in student tastes and preferences. *The Atlantic Monthly* writes, “U.S. News is always tinkering with the metrics they use, so meaningful comparisons from one year to the next are hard to make. Critics also allege that this is as much a marketing move as an attempt to improve the quality of the rankings: changes in the metrics yield slight changes in the rank orders, which induces people to buy the latest rankings to see what’s changed” ([Tierney (2013)]).

This *Atlantic Monthly* quote highlights one research question: if there are no actual changes in either student preferences or in university attribute scores over time, does a publication have an economic incentive to change the attribute score weights it uses to rank universities? A related question concerns whether the business interests of the publication and the information needs of a student seeking information are aligned. Thus, our research considers two inter-related questions: (a) If all students have the same set of attribute weights in their utility functions, is it always optimal for the publication to apply those same weights to its attribute scores in constructing its rankings? And

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2In the attribute-and-aggregate method, the ranking publication identifies attributes of importance (like “classes with fewer than 20 students,” “acceptance rate,” etc.), rates a university on the attribute, chooses a weight for the attribute, multiplies the weight times the rating and adds up the weighted sum of attribute ratings to form a university score. Universities are then rank-ordered based on the score to form a ranking.

(b) if there are no actual changes in either student preferences or in university attribute scores over
time, is it optimal for a publication to change the attribute score weights it uses to rank universities?

To investigate these questions, we construct a multi-period theoretical model of information (uni-
versity rankings) strategically transmitted by an expert (the publication) to less-informed decision
makers (students). In our model, we identify factors that provide the incentive for a rankings publi-
cation both to choose attribute score weights that do not match the students’ weights and to change
those weights, i.e., to “tinker with the metrics they use” (Tierney (2013)), from year to year. Key
features of our model are that university rankings provide information to students and that the rank-
ings themselves create prestige for highly-ranked universities, an additional attribute of importance to
students in making a choice of university.

The ordering of universities in a ranking presumably provides functional information about the
quality of the universities, whether students interpret the ranks as salient indicators of quality or use
the ranks to update (probabilistic) beliefs about the universities’ attribute scores. But a university’s
rank in a published ranking may confer prestige on the university; in our model, a prestige effect of a
published ranking is measured by the increase in utility a student experiences when the university she
chooses to attend improves its ranking. In our model, students decide whether to purchase, or bear the
time cost of viewing, the ranking. In choosing whether to do so, they compare the expected utilities
of viewing and of not viewing the ranking. In our analysis, we characterize a publisher’s optimal
ranking methodology – the weights it attaches to attribute scores in creating the overall scores to rank
universities – and compare it with two extreme ones: the student-optimal ranking methodology, which
uses student preferences over attributes to rank universities, and a uniform ranking methodology, one
that uses a uniform distribution over all possible rankings of universities to randomly select a ranking.

Our model yields three main results. First, we show that the prestige effect of university rankings
provides the incentive for a publication to use attribute score weights that do not match student weights
and therefore the publication has an incentive to rank universities in a manner that is inconsistent with
student preferences. The prestige effect pushes the publisher away from a student-optimal ranking
methodology and toward a uniform ranking methodology.

Second, we demonstrate that when a prestige effect is present, the publication optimally begins

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4 We operationalize the prestige of a university as an increasing function of its ranking, which could be interpreted as
its brand value or brand equity and is separate from the functional value.

5 In our analysis, if attribute score weights do not exist that could yield a uniform distribution over all possible rankings
of universities, then the publisher could use a random number generator based on a uniform distribution.
with student-optimal weights in its first published ranking but has an incentive to move away from those weights and toward uniform weights over time.

Third, in our dynamic analysis, we show that while a socially-optimal ranking methodology (one whose objective is to maximize the sum of students’ and publisher’s utilities) evolves toward a stable ranking of universities, the publisher’s optimal ranking methodology fluctuates over time, never converging to a stable ranking.

In Section 2 we provide a brief history of published university rankings and relate our analysis to literature about strategic information transmission. In Section 3 we set up the model. The results are in Section 4 where we first present general results about rankings in a particular period. We follow these general results by constructing examples of dynamic paths of the publisher’s optimal ranking methodologies over time. In Section 5 we investigate a socially-optimal ranking methodology and show how it differs from student-optimal and publisher-optimal methodologies. In Section 6 we discuss our results and suggest how students should cope with the current information environment and suggest modifications to university ranking methodologies that would minimize or eliminate incentives for strategic manipulations. We also comment on possible extensions of the model.

2 Product Rankings and Related Literature

2.1 Rankings

Publications that use an attribute-and-aggregate ranking methodology to rank universities, the most common approach used, collect data about university attributes, assign scores to each of the attributes, aggregate the attribute scores to determine overall scores, and then rank the products based on the overall scores. Publications in other fields use this approach as well, including Consumer Reports which ranks many products, Cook’s Illustrated which ranks cooking equipment, and the tire retailer Tire Rack which ranks tires. Although there are others, this paper focuses exclusively on attribute-and-aggregation university rankings.

University rankings first appeared in the 1870s to inform higher education scholars, professionals and government officials. Rankings gained mass appeal in 1983, over a century after

6In an alternative to the attribute-and-aggregate approach, publications determine product scores based on some type of customer satisfaction scoring or voting, and then rank products based on aggregated consumer satisfaction scores or votes. Examples include The New York Times’s bestseller list and the USA Today Coaches Poll of college football teams. Dai et al. (2014) construct an algorithm to aggregate consumer reviews on Yelp.com, which weights reviews according to their perceived value, into ranked output of searches; and develop an interesting variation of these survey- or voting-based methodologies, Avery et al. (2013) derive a revealed preference ranking of U.S. college and universities using the actual matriculation decisions made by students.
they were first introduced, when *USNWR*, using a survey of university presidents, published its first rankings of undergraduate academic quality. In 1987, *USNWR* adopted its current multidimensional ranking methodology, incorporating more objective attributes along with assessments by academic leaders of their peer institutions. The 2016 *USNWR* ranking involves attribute categories including assessment by administrators at peer institutions, retention of students, faculty resources, student selectivity, financial resources, alumni giving, and graduation performance.

When *USNWR* introduced its university rankings issue in 1983, the publication ranked the top-25 national universities and top-25 national colleges. In 1998, *USNWR* expanded its rankings of the national universities to the top-50 universities. In the 2004 ranking, *USNWR* created three categories – national doctoral universities, regional masters universities, and colleges. In its 2015 edition *USNWR* launched its best global universities ranking that rates the world’s top 400 Universities, both overall and by subject, including separate rankings for Asia and Latin America. *USNWR* also ranks graduate schools, by 11 specialties, and high schools.

University administrators, while sometimes criticizing the existence of published rankings, recognize that these rankings are publicly visible performance scorecards. For example, Hobart and William Smith College fired a senior vice president in 2000 after she failed to submit fresh data to *USNWR*, resulting in a major drop in the Colleges rank ([Graham and Thompson, 2001](#)). And Richard Beeman, Dean of the College of Arts and Sciences at the University of Pennsylvania, in a letter to the New York Times (Sept 17, 2002) commented, “I breathed a sigh of relief when my university continued to appear in the [*USNWR*] top 10.” Students use these publications to acquire information about ranks, and they affect students’ decision processes; a university’s improved rank in an influential rankings publication leads to significant increase in matriculation rates (i.e., yield) ([Griffith and Rask, 2007](#)); ([Luca and Smith, 2009](#)).

Given the importance of university rankings to students in their university decision-making process, universities choose to participate in these rankings by offering data about attributes that are of interest to those students, their (potential) customers. Each attribute-and-aggregate undergraduate university ranking includes a subset of four classes of university attributes: (a) the preferences or utilities of entering students; (b) the inputs to the university (as measured, for example, by the quality of the entering freshman class); (c) the quality of the student experience (as measured, for example, by retention and alumni giving); and (d) the graduate school and employment placement of university graduates. To the best of our knowledge, no university ranking includes all four attribute classes.
UNNWR bases its ranking on the attributes of the entering class and on measures of the student experience. The BusinessWeek ranking of undergraduate business programs includes information about student quality (e.g., average SAT scores), student experience (e.g., student perceptions of the quality of teaching), and output information (e.g., recruiter ratings and starting salaries).

The goal of each publication that ranks universities is to garner revenue by generating traffic through sales of the magazine or access to the website, with the possible associated sale of advertising, where advertising revenue is linked to traffic. To generate traffic, rankings must provide value to students, which will occur only if students expect that the information provided by the rankings will affect their decisions about which universities to apply to and attend. Rankings publications make four choices that affect student value: prices for access and advertising; the attribute scores to include in their rankings; their investments in the measurement of attribute scores; and the function used to aggregate attribute scores into a ranking. Our analysis of attribute-and-aggregate rankings is about a publication’s strategic choice of the aggregation function – the weights it uses in aggregating attribute score weights into overall scores. In our analysis, the ranking methodology is equivalent to selecting attribute score weights.

USNWR regularly changes its ranking methodology – the attribute score weights – it uses to rank universities. In 2016, for example, the publication, which uses survey results from peer institutions and guidance counselors in determining its rankings, moved to using multiple years of the survey results.7

2.2 Expertise, The Value of Rankings, and Information Transmission

More generally, our model is about the strategic transmission of information (SIT) by an informed expert (i.e., sender, ranking publication in our context) to an uninformed or lesser-informed decision maker (i.e., receiver, student-consumer in our context) who receives the information prior to making a decision (i.e., choosing a university to attend in our context) (see Crawford and Sobel 1982, Chakraborty and Harbaugh 2010 for example). Models of this process, including ours, are concerned with the same question: Why does an informed expert choose to misrepresent information to a decision maker?8 One reason presented in the literature is that the information conveyed by the sender influences the receiver’s decision, which in turn affects the sender’s utility. For example, suppose the

8 Battaglini (2002) demonstrates that in a scenario with multidimensional information transmission, a receiver of information (e.g., consumers or students) can design incentive schemes for multiple senders (e.g., product or university experts) to fully reveal information. However, in the rankings context (for products or universities), consumers or students do not design incentive schemes.
sender is an expert weather forecaster who owns shares in an umbrella manufacturer. The forecaster may overstate the probability of rain in order to increase the likelihood that viewers of his forecast will purchase umbrellas.

In the SIT models cited above, the factors that cause an expert to misrepresent information are not the same as those that cause a publisher to offer misleading advice to students through its ranking; the institutional features of the publisher-student relationship differ significantly from those in the strategic communication models. Thus, the strategic communication literature cannot be applied to the university ranking and student choice problem for at least three reasons.

First, in the SIT literature, the expert misreports information to the decision maker in order to influence the action chosen by the decision maker. The expert does so because its utility is affected by the decision made by the receiver. However, in the context of university rankings, the expert (i.e., ranking publication) is not concerned with the decision maker’s (i.e., student’s) choice of university to attend. Second, in the SIT literature, the receiver’s outside option utility is independent of any actions that would be taken in the relationship. However, in our context, when a prestige effect is present the students’ outside option utilities (i.e., the students’ utilities of not viewing a ranking) are affected by the publisher’s ranking. Third, in the SIT literature, the receivers do not incur a cost for accepting the sender’s advice, and therefore optimally do so. In our context, viewing a ranking incurs a cost, whether in terms of money or time; students for whom the cost is sufficiently low and the value high will chose to view the rankings while others will not. Thus, the publication’s optimal ranking methodology affects the students’ decisions on whether or not to view the ranking.

Because the SIT models are inappropriate for our context, we develop a new model. In our model: (a) the students’ expected utilities of attending universities (after viewing the ranking) are affected by the publisher’s ranking, but the publisher’s profit is not affected by the students’ choices of universities to attend; (b) when a prestige effect is present, the students’ utilities for attending universities are affected by their ranks, whether or not the students view the ranking; and (c) students experience different costs and values of viewing the ranking, and only some choose to view.

The distinction between the information provided by university rankings and the prestige they create is key in our analysis; university rankings are like advertising in the sense that advertising also can inform consumers about product attributes (Stigler 1961; Nelson 1974; Butters 1977) and can affect a product’s prestige (Galbraith 1976; Becker and Murphy 1993; Ackerberg 2001).

Consumers value products for more than their functional value; they value them for their meaning
and signaling value as well (Fournier, 1998). Luxury goods are developed to provide such signal value, where the functionality of the product may be of less importance to the consumer than the prestige that acquiring the product conveys (Han et al., 2010). College and university selection has some characteristics in common with the consumption of luxury goods, where simply acquiring the good signals status. If employers use university ranks as measures of the human capital of graduates, then students have an economic reason to attend better-ranked universities, independent of the quality of the education there. In addition, there are the bragging rights for those attending highly ranked schools. Just as advertising can directly affect the utility that a consumer derives from consuming the advertised product (Becker and Murphy, 1993), a university’s rank can directly affect the utility a student derives from attending the university.

There are at least two ways that university rankings can provide information to students. First, a ranking’s publisher knows attribute scores while most students do not. Basing a ranking on attribute scores therefore provides information to students about the attribute scores. Second, a publisher may have expertise in evaluating the relative importance of various attributes. In the auto tire world, technical managers at the tire retailer Tire Rack, which publishes an attribute-and-aggregate tire ranking, presumably know far more than most consumers about the relative importance of tire attributes. Hence, in the context of universities, students may rely on publishers to effectively set their preferences. In our model, a ranking provides aggregated information about university attributes to students who evaluate products based on attribute scores, and a ranking also provides information to students who use the salience of the ranking to evaluate universities.

If bragging rights cause a prestige effect and a student learns the rank of her university after choosing to attend, then she experiences a prestige effect even though she did not view the ranking. If there are labor market premiums from attending better-ranked universities, then she experiences an increase in utility even if she never learns her university’s rank.

The prestige effect, not addressed in the SIT literature, is critical to our analysis. If a publisher chooses weights for attribute scores that do not match student preferences, then the publisher lowers the students’ expected utilities of viewing the ranking. However, when a prestige effect is present, the publisher also lowers the expected utilities of not viewing the ranking (by increasing the students uncertainty about the actual ranking), possibly by an amount sufficient to raise the net expected utility of viewing versus not viewing its ranking. In contrast, when a prestige effect is not present, the publisher, in choosing a ranking methodology, does not affect the utilities of students who do not view...
the ranking. In that case the publication’s objective reduces to maximizing the expected utilities of the students who view the ranking, and it does so by choosing a student-optimal ranking methodology.

Our model shows that a prestige effect causes a publication not to act in the best interest of students. While the popular press and universities have criticized rankings publications for changing the ranking methodology to yield changes in the rank ordering, they have not isolated the source of this behavior – the prestige effect – that we do here.

3 Model Setup

We develop a multi-period model of the relationship between a monopoly publication that ranks universities and students (e.g., high school seniors considering attending a university) who choose whether or not view the ranking and use it in their decisions about which universities to attend. Our model characterizes universities by their attribute scores such as the SAT scores of their entering classes, the sizes of their classes, and their graduation rates. The publication offers only an overall ranking of universities and does not report attribute scores.

In our model students know both the university attributes that are important to them and the weights those attributes play in their utility functions. We assume the publication is an expert about universities in the sense that it knows the attribute scores of each university, while students do not. Students form prior probabilistic beliefs about attribute scores and, if they view the publication’s ranking, update these beliefs about university attributes based on the universities’ ranks in the publication; the students are Bayesian decision makers.

Our model includes two ways in which a university’s rank in the publication can affect the expected utilities of students who will be attending universities. First, as described above, students use ranks to update probabilistic beliefs about university attribute scores. Second, a university’s rank affects student utility directly due to the prestige created by a (good) rank, as described in the previous section.

3.1 Timing

Our model has one rankings publication and $n$ universities, each operating in every period, $t, t = 1, 2, ...$. During each period $t$, the information states and order of events are: (1) At the beginning of the period, the publication and students are uncertain about the universities’ attribute scores. As we describe below, the publication knows some details about the preferences of the students but is uncertain about
others. The publication and the students know the number of students who viewed the ranking in
the prior period. (2) The publication chooses its ranking methodology – the weights it attaches to
attribute scores. (3) The publication learns the attribute scores and then ranks the universities.
(4) The students choose whether to view the ranking. Students who view the ranking update their
probabilistic beliefs about the attribute scores. (5) The students choose which universities to attend.

3.2 Characterizing the Universities

Assumption 1. University $i$, $i = 1, ..., n$, in period $t$, $t = 1, 2, ...$ is characterized by $m$ attributes,

$$a_i^t = (a_{i1}^t, ..., a_{im}^t) \in \mathbb{R}^m.$$  

Let $a^t = (a_1^t, ..., a_n^t)$ denote a profile of the attributes of all universities and $\mathcal{A}$ represent the set of
all possible attribute profiles. Although the publication ranks universities and the students evaluate
them, the universities take no actions and therefore are not active in our model.

3.3 The Publication’s Ranking Methodology

A ranking in period $t$ is denoted as $r^t$, an ordered $n$-tuple in which the universities are ranked from
the best in position 1 through the worst in position $n$. University $i$’s position in the period-$t$ ranking
$r^t$ is represented by $r_i^t$. The set of all possible rankings is denoted as $\mathcal{R}$.

We assume the publication uses a linear attribute-and-aggregate ranking methodology in which each
university’s aggregated score is a linear weighted sum of its attribute scores, a ranking methodology
we consider because it is used by popular university rankings like USNWR. Let $w^t \equiv (w_1^t, ..., w_m^t)$
denote the publication’s period-$t$ list of attribute weights.

Assumption 2. In the publication’s linear attribute-and-aggregate ranking methodology, university
$i$’s period-$t$ aggregated score is a weighted sum of its attribute scores, $\sum_{j=1}^{m} w_j^t a_{ij}^t$. The publication
ranks universities according to the aggregated scores: If $\sum_{j=1}^{m} w_j^t a_{ij}^t > \sum_{j=1}^{m} w_j^t a_{i'j}^t$, then $r_i^t < r_{i'}^t$.

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9The publication sets its ranking methodology before learning the attribute scores. Furthermore, the publication
reports neither its ranking methodology nor the universities’ attribute scores, consistent with the fact that, in practice,
rankings publications typically offer only an outline of their methodologies and either an incomplete list of, or approxi-
mate, attribute scores. While the publication does not report its period $t$ ranking methodology to the students, because
students have complete information about the publication’s objective function, they are able to infer the publication’s
equilibrium ranking methodology.

10Despite the loss of generality in assuming a linear ranking methodology, our ranking methodology does permit “convex
preferences.” For example, using our notation, two attribute scores could be the natural log of academic spending per
student and the average SAT score of an entering class. If so, the iso-aggregate-score surface would be strictly convex in
academic spending per student and in average SAT score.
3.4 The Publisher’s Objective Function and Decision Problem

Online rankings, most notably *USNWR* and *BusinessWeek*, offer free access to their rankings, and therefore they do not generate revenues by selling access to their rankings. Rather, they generate revenues either from advertisements placed on their rankings pages or by selling subscriptions to detailed information about colleges and universities, where the subscription pages are linked to their rankings pages. In either case, a publisher’s revenue is increasing in the number of views of its ranking.

Because our analysis is about a publisher’s design of its ranking methodology and not about the pricing of either click-through advertisements or the pricing of subscriptions to detailed information about universities, we assume a simple, linear relationship between the number of views of a ranking and the revenues generated by the ranking. In this linear relationship, the revenue per view is constant, which means that if the publication generates revenues by click-through ads, the rate per click-through is constant in the number of clicks, and if the publication generates revenues by selling detailed information about colleges and universities, the demand for this detailed information provided by the publisher is perfectly price elastic. To simplify our notation, we normalize the revenue per student who views a period-$t$ ranking at 1.

The costs of publishing an online ranking are almost entirely fixed: developing the publication’s ranking methodology, collecting the necessary data, and designing the presentation of the online content. Costs that vary with the number of views of the ranking are negligible so we set the marginal cost to the publisher of the number of views at zero. More formally:

**Assumption 3.** In our multi-period model, the publisher’s revenue generated from its ranking publication equals the sum over all periods of the number of students who view its ranking and the publisher chooses attribute weights in each period to maximize its revenue.

The number of students who view the ranking in period $t$, which we denote as $s^t$, is a function of the publication’s period-$t$ ranking methodology, $w^t$, and also of the number of students who viewed the ranking in period $t-1$, $s^{t-1}$. As we discuss subsequently, when a prestige effect is present, student utility in period $t$ of attending a university, of viewing the ranking, and of not viewing the ranking, depends on the number of students who viewed the ranking in $t-1$, $s^{t-1}$.

The publisher’s revenue is:

$$\sum_{t,t=1,2,...} s^t.$$
The publication’s objective is to choose each period’s attribute weights to maximize that revenue: the sum over time of the number of students who view its rankings:

$$\max_{w^t, t=1,2,...} \sum_t s^t(w^t; s^{t-1}). \quad (1)$$

The goal of our analysis is to characterize optimal attribute weights, $w^t, t = 1, 2, \ldots$, that solve the publisher’s maximization problem, \(\blacklozenge\). In our analysis of the publisher’s choice of $w^t, t = 1, 2, \ldots$, the weights (which define the publisher’s ranking methodology) affect student utility and choices, including the choice of whether to view the ranking, central to our model.

3.5 The Students

**Assumption 4.** In each period $t$ a new unit-mass batch of students enters the education market. Furthermore, each student has been admitted by each university.

Because each student attends a university for only one period, a period-$t$ student is concerned with attributes of the universities, $a^t$, and the university ranks, $r^t$, in only period $t$.

3.5.1 Students’ Utilities

**Assumption 5.** Each student’s utility of attending university $i$ can be represented by a linear compensatory function.

As noted, a ranking provides information about the experience students can expect at a university and also may carry prestige. A student forms her utility by assigning a weight to each individual attribute score, a weight to the aggregated attribute score, and a weight to the university’s ranking. We let $\gamma_j^t$ be the weight a student in period-$t$ assigns to attribute $j$ and $\alpha^t$ be the weight the student assigns to the aggregated attribute score. The component of a student’s utility from attending university $i$ that includes the attribute scores is $\alpha^t \sum_j \gamma_j^t a_{ij}^t$.

The prestige component of a student’s utility function is the product of three terms. First, we denote the importance of the prestige of attending university $i$ to a student by the weight $\beta^t$. Second, we capture the fact that the effect of the prestige of a university’s rank on a student’s utility is increasing in the popularity of the ranking by including a component $g(s^t s^{t-1})$, which is strictly increasing in $s^{t-1}$, in a student’s utility function.\[11\] Also, the function $g$ permits a multiplier effect from views of the publisher’s ranking associated word-of-mouth advertising about the ranking on the magnitude of the

\[11\]To simplify, we specify $g$ as a function of the popularity of the ranking in only period $t - 1$, $s^{t-1}$.
Third, we specify that a student’s utility of attending a university is strictly decreasing in the university’s rank (given the top university is ranked number 1). The strictly decreasing function $R(r_i^t)$ captures the effect of university’s $i$’s rank on utility. With these three components of the effect of prestige on utility, the effect of the prestige of attending university $i$ on a student’s utility is 

$$\beta^t g(s^{t-1})R(r_i^t).$$

Hence, a student’s utility of attending university $i$ is:

$$U_i^t = \alpha^t \sum_j \gamma_j^t a_{ij}^t + \beta^t g(s^{t-1})R(r_i^t).$$

(2)

**Assumption 6.** Students differ on the weights they assign to the information provided by the ranking, $\alpha^t$, and the prestige of the rankings, $\beta^t$, in their utility functions. However, all students attach the same weight to each attribute $j$, $\gamma_j^t$.

The values of $\alpha^t$ and $\beta^t$ in the period-$t$ student population are distributed according to the population distribution functions $H_{\alpha}^t(\alpha^t)$ and $H_{\beta}^t(\beta^t)$ and the corresponding density functions $h_{\alpha}^t(\alpha^t)$ and $h_{\beta}^t(\beta^t)$ respectively. While students differ by the importance they attach to attribute scores overall and to university ranks, all students have the same weights they attach to specific attribute scores, $\gamma^t = (\gamma_1^t, ..., \gamma_m^t)$, in their utility functions. The publication knows $H_{\alpha}^t$, $H_{\beta}^t$, and $\gamma^t = (\gamma_1^t, ..., \gamma_m^t)$.

### 3.5.2 Student Probabilistic Beliefs

**Assumption 7.** All period-$t$ students have prior probabilistic beliefs about each university’s attribute scores, and then, if they view the publisher’s period-$t$ ranking, they update these beliefs using Bayes’ rule.

At the beginning of period $t$, each student develops probabilistic beliefs as follows. Let $p_{ij}^t(a_{ij}^t)$ represent the probability density function for university $i$’s $j$’th attribute score, $a_{ij}^t$. By assuming independent distributions on attribute scores: we can write the probability density function on university $i$’s attribute profile $a_i$, which we denote by $p_i^t(a_i^t)$, as $p_i^t(a_i^t) = p_{i1}^t(a_{i1}^t) \cdots p_{im}^t(a_{im}^t)$, and we can write the probability density function on the profile of attributes of the $n$ schools $a^t$, which we denote

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12 This specification is consistent with much work in the area including Chevalier and Mayzlin (2006). See Peres et al. (2010) for a review of the related literature.

13 The absolute magnitudes of $\alpha^t$ and $\beta^t$ help determine whether a student chooses to view the ranking. So, we do not set $\beta^t = 1 - \alpha^t$.

14 We assume that all students have the same $\gamma^t$ so that we can examine whether the publication chooses to rank universities in a manner that is consistent with the students’ preferences – the relative importance of each attribute score.
by $p(t', a')$, as $p(t') = p_1(t_1') \cdots p_n(t_n')$. Note that for any $t$ and $t'$, $a(t)$ and $a(t')$ are independently but not necessarily identically distributed. The density functions may change over time, indicating changes in a distribution of a university’s quality.

Based on these prior beliefs about attribute scores and also on the publisher’s equilibrium strategy, all period-$t$ students form probabilistic beliefs about the publisher’s ranking, $r(t)$. Students who have not (yet) viewed the ranking $r(t)$ believe it is distributed, conditional on an attribute profile $a(t)$ and the publisher’s period-$t$ equilibrium ranking methodology $w(t)$, as $q(t|a(t); w(t))$. Because students cannot observe attribute scores, their probabilistic belief about $r(t)$, conditional on only $w(t)$, is relevant to their decision making as follows:

$$q(t|w(t)) = \int_{a(t)} \cdots \int_{a(t)} q(t|a(t); w(t))p_1(t_1) \cdots p_n(t_n) \, da(t) \cdots da(t).$$

(3)

We write the conditional probability of $a(t)$, conditional on $r(t)$ and $w(t)$, updated using Bayes’ rule, as $p(t|a(t); w(t))$.

### 3.5.3 Student Expected Utility and Decision Rules

**Assumption 8.** A student views the publisher’s ranking if and only if her expected utility of viewing is greater than her expected utility of not viewing.

If a student views the ranking, then she uses the publication’s ranking $r(t)$ and the updated probability distribution of attribute scores, $p(t|a(t); w(t))$ for each $a(t)$, to calculate the expected utility of attending each university $i$:

$$E[U_{i,\text{View}}^t] = \alpha(t) \int_{a(t)} \cdots \int_{a(t)} p(t|a(t); w(t)) \sum_j \gamma_j a(t) j \, da(t) \cdots da(t) + \beta(t) g(s(t-1))R(r_i(t)).$$

(4)

Given the ranking $r(t)$, she chooses to attend the university with the greatest expected utility, as expressed in (4).

To calculate the expected utility of viewing the ranking, for each possible $r(t)$, she determines the university with the greatest expected utility of attending. She then uses the equilibrium probability distribution over ranks, $q(t|w(t))$ as specified in (3), to calculate her expected utility of viewing the ranking:

$$V_{\text{View}}^{t} (w(t); s(t-1)) \equiv \sum_{r(t) \in R} q(t|w(t)) \max_i E[U_{i,\text{View}}^t].$$

(5)

---

We examine deterministic ranking methodologies in the sense that for each $a(t)$, $q(t|a(t); w(t)) \in \{0, 1\}$. 

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If a student chooses not to view the ranking, then she uses the publisher’s equilibrium conditional probability of rankings, $q^t(r^t|a^t; w^t)$, and the prior probability distribution about attribute scores, $p^t(a^t)$, to calculate the expected utility of attending each university $i$:

$$E[U_{i,Not}^t] = \int_{a_{11}^t} \cdots \int_{a_{nm}^t} p^t(a^t) \left( \alpha^t \sum_j \gamma_{t}^j a_{ij}^t + \sum_{r^t \in R} q^t(r^t|a^t; w^t) \beta^t g(s^{t-1}) R(r^t_i) \right) da_{11}^t \cdots da_{nm}^t. \quad (6)$$

Note that if the student views the ranking, then her expected utility of attending a university depends in part on the university’s actual rank. However, if she does not view the ranking, then her expected utility depends in part on the expected rank of the university.

A student’s expected utility of not viewing the ranking is then:

$$V_{Not}^t(w^t; s^{t-1}) = \max_i E[U_{i,Not}^t], \quad (7)$$

where $E[U_{i,Not}^t]$ is specified in (6).

A student chooses to view the publication if and only if $V_{View}^t(w^t; s^{t-1}) - c \geq V_{Not}^t(w^t; s^{t-1})$. We let

$$I^t(w^t; s^{t-1}) = \begin{cases} 
1 & \text{if } V_{View}^t(w^t; s^{t-1}) - c \geq V_{Not}^t(w^t; s^{t-1}), \\
0 & \text{otherwise} 
\end{cases} \quad (8)$$

indicate whether a student in period-$t$ views the publication. The number of students who view the period-$t$ publication is:

$$s^t(w^t; s^{t-1}) = \int \int I^t(w^t; s^{t-1}) h^t_\alpha(\alpha^t) h^t_\beta(\beta^t) d\alpha^t d\beta^t.$$

### 4 Model Analysis

Our goal here is to demonstrate that the prestige effect provides an economic incentive for the publisher to (a) set attribute score weights that differ from the students’ weights; and to (b) change its attribute score weights over time simply to change rankings year to year and thereby induce more students to view them.

Our analysis begins in Section 4.1 with Lemma 1 which establishes that in solving the publisher’s optimal ranking methodology for each period of the dynamic model, we need only consider the views in $t - 1$ and need not look ahead to future equilibrium play between the publication and students. This no-look-ahead result simplifies our dynamic analysis.

In Section 4.2, Lemma 2 and Theorem 1 identify sufficient conditions for which the publisher’s
optimal ranking methodology in period $t$ is student-optimal and sufficient conditions for which it is uniform (essentially random).

In Section 4.3 we investigate the dynamics of the publisher’s optimal ranking methodology. Lemma 3 establishes that under certain stability conditions, the number of students who view the ranking grows over time. With growing popularity of the ranking, the prestige effect of the ranking grows as well. Theorem 2 demonstrates that with a growing prestige effect, the publication begins in period 1 by using the student-optimal ranking methodology as its first published ranking. Over time, the publication sees an increase in the number of students who view its ranking and its prestige effect grows. As its prestige effect grows the publisher moves its ranking methodology farther from the student-optimal and closer to the uniform ranking methodology.

4.1 Preliminary Result

The following preliminary result simplifies our analysis.

Lemma 1. If

$$(w_1^*, w_2^*, ...) = \arg\max \sum_t s^t(w^t; s^{t-1}),$$

then for each $t$,

$$w_t^* = \arg\max s^t(w^t; s^{t-1}).$$

Lemma 1 demonstrates that in our dynamic model, we can calculate the optimal period-$t$ ranking methodology with no look-ahead. That is, to calculate the optimal ranking methodology in period-$t$, the publication takes the number of views from $t - 1$ and then sets $w^t$ to maximize $s^t$ with no regard to the effect of $w^t$ on future views, $s^{t+1}, s^{t+2}, ...$.

4.2 The Methodology in Period $t$

In analyzing the period-$t$ ranking methodology, we show that the prestige effect of university rankings provide the incentive for a publication to use attribute score weights that do not match student weights and therefore to rank universities in a manner that is inconsistent with student preferences. We consider two special cases: one in which no students experience a prestige effect; and the other in which all students experience only a prestige effect and are not directly affected by university attributes.

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16 The proofs of all lemmas and theorems are in Appendix A.
For these two cases Theorem 1 demonstrates that in any period $t$ of our dynamic model, the optimal ranking methodology is either student-optimal or uniform. We define the student-optimal ranking methodology as the one in which publication uses the student’s preferences in determining each university’s score and therefore ranks the universities according to $w^t = \gamma^t$. We define the uniform ranking methodology as the one in which the publication uses a uniform distribution to randomly select the positions of the schools in its ranking as follows: for each $r$, $q_R(r) = \frac{1}{m}$.

We begin with Lemma 2 which is used in the proof of Theorem 1.

**Lemma 2.** Consider period $t$. Suppose the ranking publication has viewers in period $t-1$, $g(s^{t-1}) > 0$.

(a) If the students are unconcerned with the prestige of university ranks ($H^t_{\alpha}(0) < 1$ and $H^t_{\beta}(0) = 1$), then in choosing its ranking methodology to maximize $s^t(w^t; s^{t-1})$, the publisher maximizes $V_{View}(w^t; s^{t-1})$.

(b) If the students are concerned with only the prestige of the university ranks ($H^t_{\alpha}(0) = 1$, $H^t_{\beta}(0) < 1$), then in choosing its ranking methodology to maximize $s^t(w^t; s^{t-1})$, the publisher minimizes $V_{Not}(w^t; s^{t-1})$.

We now characterize the publication’s optimal period-$t$ ranking methodology, $w^t$, that maximizes $s^t$.

**Theorem 1.** Consider period $t$. Suppose the ranking publication has viewers in period $t-1$, $g(s^{t-1}) > 0$.

(a) If the students are unconcerned with the prestige of university ranks ($H^t_{\alpha}(0) < 1$ and $H^t_{\beta}(0) = 1$), then the student-optimal ranking methodology ($w^t = \gamma^t$) maximizes $s^t$.

(b) If the students are concerned with only the prestige of the university ranks ($H^t_{\alpha}(0) = 1$, $H^t_{\beta}(0) < 1$), then the uniform ranking methodology, $q_R(r) = \frac{1}{m}$, maximizes $s^t$.

Theorem 1 demonstrates that if the students are not concerned with prestige, then the publication optimally uses the students’ preferences to determine its attribute weights in establishing the scores of the universities and therefore in ranking the universities. With no prestige effect and if student preferences remain the same over time, the publication’s optimal ranking methodology – the student-optimal ranking methodology – will remain unchanged over time. The variability of the rankings over time then would not depend on changes in the attribute weights the publication optimally uses to determine its rankings, but rather on the variability of the universities’ attribute scores over time.
Therefore, if these attribute scores are stable (or variable), then the rankings will be stable (or variable) as well.

In contrast, if students are concerned only with prestige, then the publication optimally uses the uniform (or random) ranking methodology in each period to select attribute weights; hence those weights will vary over time, leading to variations in ranking. To understand the mechanics of the publisher’s choice of ranking methodology, note that each student wants to attend the top-ranked school. If the student views the publication, then she can choose the top university. But, if she does not view the publication, then she may make a mistake and attend a lesser-ranked university. Therefore, to maximize the net utility to the students of viewing the ranking, the publication wants to maximize the probability that a student who does not view the published ranking makes a university selection mistake. The publication does so by selecting attribute weights that put each university in each position with equal probability.

While Theorem 1 offers insight into two special cases, it does not address the more realistic cases in which students are concerned both with the information a publisher’s ranking provides and the prestige it creates. We examine these cases in a multi-period context.

4.3 Dynamics

We begin our dynamic analysis in Lemma 3 by demonstrating that in a stable environment (one where the distributions of the weights $\alpha$ and $\beta$ and the distribution of the universities’ attribute scores remain unchanged over time), then the number of students who view the ranking grows over time. With an increase in the number of students who view the ranking over time, the prestige effect of the ranking grows as well. Following Lemma 3 in Theorem 2, we demonstrate that in this stable environment, the publication begins by using the student-optimal ranking methodology. Over time, as the number of viewers grows, the publication has an incentive to move its ranking methodology further from the student-optimal methodology and closer to the uniform ranking methodology. Therefore, even in a stable environment, which should be most conducive for an unchanging ranking methodology, the publication has an incentive to change its ranking methodology over time.

**Lemma 3.** If for each $t'$ and $t''$, students have the same weights for each attribute score ($\gamma_{t'} = \gamma_{t''}$), the weights ($\alpha^{t'}, \beta^{t'}$) and ($\alpha^{t''}, \beta^{t''}$) are identically distributed ($H_{\alpha}^{t'} = H_{\alpha}^{t''}$ and $H_{\beta}^{t'} = H_{\beta}^{t''}$), and the attribute scores are identically distributed ($p_t^{t'} = p_t^{t''}$), then for any $t$, $s^t > s^{t-1}$.

We use Lemma 3 in Theorem 2 to demonstrate that, due to the growth in the number of students
who view the ranking and the publication’s increasing influence over time, the publication changes its ranking methodology over time. A technical, but important, issue is that the effect of the increase in the number of students who view the ranking on the publication’s choice of ranking methodology can be partitioned into two effects. First, the prestige effect becomes relatively more important as the number of viewers grows. Second, as the number of viewers grows, the cutoff in the \((\alpha, \beta)\) space, between the students who view the ranking and the students who do not, shifts. As a result of this shift, there may be a difference between the distributions of \(\alpha\) and \(\beta\) around the new cutoff and the distributions of \(\alpha\) and \(\beta\) around the old cutoff. If the distributions of \(\alpha\) and \(\beta\) around these two cutoffs differ, then the effect of a change in the ranking methodology causes differing numbers of students to switch between viewing and not viewing the ranking. As a result, this distributional effect causes a change in the marginal value to the publisher of changing its ranking methodology, inducing the publication to change its ranking methodology.

Our goal in Theorem 2 is to consider a case that is not subject to this distributional effect (the second effect) thereby isolating the greater importance of the prestige effect (the first effect) associated with the increase in the number of students who view the ranking. For uniform distributions on \(\alpha\) and \(\beta\), the distributions \(\alpha\) and \(\beta\) around the cutoff between viewing the ranking and not viewing the ranking is independent of the position of the cutoff. Therefore, in Theorem 2 we consider uniform distributions on \(\alpha\) and \(\beta\)\(^{17}\).

Note that in Theorem 2 we examine the simplest possible case of two universities and two attributes. In our analysis of this two-attribute case, without loss of generality, for each \(t\), we normalize \(\gamma_1^t = 1\) and \(w_2^t = 1\). With \(w_2^t = 1\), in each period \(t\), the publication chooses only \(w_1^t\); and we let \(w_1^{uniform}\) denote the weight that implements a uniform ranking methodology.

**Theorem 2.** Consider the case with two universities, \(n = 2\), with two attributes, \(m = 2\). In each period \(t\), the student weights are uniformly distributed, \(H_\alpha(\alpha^1) = H_\alpha(\alpha^2) = \ldots = \frac{\alpha^t}{\pi}\) and \(H_\beta(\beta^1) = H_\beta(\beta^2) = \ldots = \frac{\beta^t}{\beta}\); and in each period \(t\), the attribute scores of the universities are distributed according to \(p^1(\alpha^1) = p^2(\alpha^2) = \ldots\). If \(\gamma_1 > w_1^{uniform}\), then for each \(t\) and \(t - 1\), \(w_1^t < w_1^{t-1} < w_1^* = \gamma_1\); and if \(\gamma_1 < w_1^{uniform}\), then for each \(t\) and \(t - 1\), \(w_1^t > w_1^{t-1} > w_1^* = \gamma_1\).

Theorem 2 shows that as the ranking increases in popularity and, as a result, the prestige effect grows.

\(^{17}\)The response by the publisher to a change in the importance of prestige is analogous to a substitution effect and the response to a change in the distributions of \(\alpha\) and \(\beta\) around the cutoff is analogous to an income effect. Considering uniform distributions on \(\alpha\) and \(\beta\) therefore permits us to isolate a changing-prestige/substitution effect.

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over time, the publication’s optimal ranking methodology also changes over time. The attribute score weights in the optimal ranking methodology move away from the student-optimal ranking methodology and toward the uniform ranking methodology. We illustrate this finding through two dynamic numerical examples next.\footnote{18}

Each example includes a prestige effect that grows as the number of students who view the ranking grows. Two factors cause the publication to change its ranking methodology over time: a change in the number of students who have view the ranking (both examples) and a change in student preferences (Example 2).

Each of the examples has two universities and each university is characterized by the scores of attributes 1 and 2. We normalize the weight that the publication attaches to attribute 2 at $w^2_t = 1$ and examine the publication’s optimal weight it attaches to attribute 1 in each period $t$, $w^1_t$. In each case, because the publication has no viewers in period 0 and therefore has no prestige in period 1, by Theorem 1, the publication begins in period 1 by setting the student-optimal weight attached to attribute 1, $w^1_1 = \gamma^1_1$. The details of each of the examples are in Appendix B.

4.3.1 Dynamic Example 1

As displayed in Exhibit 1, the publication begins in period 1 by setting the student-optimal ranking methodology, $w^1_1 = \gamma^1_1$, which in this example equals 2. As the number of views of the publication’s ranking $s^t$ grows over time, the publication becomes more influential to students in their evaluation of universities. As the publication gains in influence, it reduces university 1’s probabilistic dominance of the rankings by reducing the probability university 1 is ranked first by about eight percent. Also as the number of views grow, the publication changes its ranking methodology by reducing $w^1_t$ through period 8, where it converges to $w^8 = w^9 = 1.64$.

Exhibit 2 displays the ranking, whether university 1 or university 2 is ranked first, in terms of the attribute scores and the ranking methodology used by the publication. It also shows the loss in student utility of viewing the ranking from the publication using a ranking methodology that is not student optimal, i.e., $w^1_t \neq \gamma^1_1$.

In this example, if the publication were to place equal weights on the two attributes (i.e., $w^1_t = 1$), then the publication would be implementing a uniform ranking methodology. However, because

\footnote{18}These numerical examples specify distributions for $\alpha$ and $\beta$ that are not uniform. Therefore, changes in the relative importance of prestige and the distributions of $\alpha$ and $\beta$ around the cutoff line between viewing and not viewing the ranking both affect the publication’s choice of ranking methodology.
the students have a greater preference for attribute 1 (i.e., $\gamma_1^t = 2$), the student-optimal ranking methodology favors school 1 and is different from this uniform ranking methodology. In fact, under the student-optimal ranking methodology, school 1 is ranked first with probability $q^t((1, 2); w_1^t = \gamma_1^t = 2) = 0.825$.

### 4.3.2 Dynamic Example 2

In this example, we retain the numerical values specified in Example 1 with the one exception: we set the initial value of $\gamma_1^1$ at 2.1 and then decrease the value beginning in period 6 to $\gamma_1^6 = 2.0$. As seen in Exhibit 3, the publication begins in period 1 by using the student-optimal ranking methodology, $w_1^1 = \gamma_1^1 = 2.1$. Then, as the number of views grows, the publication moves its ranking methodology from being student-optimal toward the uniform ranking methodology. Responding to the period-6 shock to the students’ preferences, the publication cuts its attribute-1 weight from $w_1^5 = 1.89$ to $w_1^6 = 1.77$. However, because the publication has an established number of views (i.e., $s_5 = 0.808$), it does not set $w_1^6$ equal to the student-optimal weight $\gamma_1^6 = 2.0$. Rather, it sets this weight closer to the weight of the uniform ranking methodology.

In Example 2, changes in the number of views and student preferences drive the shifts in ranking methodology, which are all departures from the student-optimal mechanism. Furthermore, the shock to student preferences leads not only to a one-time shift in ranking methodology, but also to a sequence of changes in ranking methodology.\(^{19}\)

In these two examples, the publication sets the student-optimal ranking methodology in the first period and then changes its ranking methodology over time. We demonstrate in the next section that these changes in ranking methodology reduce the social welfare of market participants.

### 5 A Socially-Optimal Methodology

If the ranking has a prestige effect, then in the cases we identified, the publisher chooses ranking methodologies that are not optimal for students who view the ranking. But the question remains about whether either the publisher’s optimal ranking or the student-optimal ranking is socially optimal.

A socially-optimal ranking methodology must consider the effects of the ranking not only on the publisher and the students who view the ranking, but also (when a prestige effect is present) on the public.\(^{19}\)

\(^{19}\)Qualitatively similar results hold for a wide range of other numerical examples, not reported here due to space constraints.
students who do not view the ranking. These non-viewing students must be considered because a prestige effect creates an externality for them, which a socially-optimal ranking should account for.

We first write a social welfare function as the sum of the students’ expected utilities who view the ranking, the students’ expected utilities who do not view the ranking, and publisher’s expected profit. Theorem 3 examines the welfare of the students who do not view the ranking. The theorem demonstrates that those students see the university ranked first as the one that gives them the greatest expected utility in terms of the attribute scores. Given unchanging preferences (i.e., weights attached to attributes in utility functions) and unchanging attribute score distributions, the students who do not view the ranking would want this university to be ranked first in every period. In a dynamic numerical example we show that a socially-optimal ranking methodology moves toward a stable ranking of universities, while the publisher’s optimal ranking methodology moves away from a stable ranking.

The utilities or profits of the parties affected by the ranking are the following. The aggregated utilities of the students who view the ranking, including the cost \( c \) of viewing, is:

\[
\int \int (V_{\text{View}}^t(w^t; s^{t-1}) - c)I^t(w^t; s^{t-1})H_\alpha^1(\alpha^t)H_\beta^1(\beta^t)d\alpha^t d\beta^t. \tag{9}
\]

The external effect of the ranking on the utilities of the students who do not view the ranking is:

\[
\int \int V_{\text{Not}}^t(w^t; s^{t-1})(1 - I^t(w^t; s^{t-1}))H_\alpha^1(\alpha^t)H_\beta^1(\beta^t)d\alpha^t d\beta^t. \tag{10}
\]

If \( c \) represents a monetary price of viewing the ranking and the marginal cost of selling an additional view is zero, then the publisher’s profit (not including the fixed cost of developing the ranking) is:

\[
c \int \int I^t(w^t; s^{t-1})H_\alpha^1(\alpha^t)H_\beta^1(\beta^t)d\alpha^t d\beta^t. \tag{11}
\]

A socially-optimal period-\( t \) ranking methodology maximizes the sum of (9), (10), and (11):

\[
\int \int (V_{\text{Read}}^t(w^t; s^{t-1})I^t(w^t; s^{t-1}) + V_{\text{Not}}^t(w^t; s^{t-1})(1 - I^t(w^t; s^{t-1}))H_\alpha^1(\alpha^t)H_\beta^1(\beta^t)d\alpha^t d\beta^t, \tag{12}
\]

which equals the aggregate utility of all students, not including the cost to those who view the ranking. We denote a socially-optimal period-\( t \) ranking methodology that maximizes (12) as \( w^t_\circ \).

**Theorem 3.** Consider period \( t \) of a two-university case, \( n = 2 \). Suppose

\[
\int a_{11} \cdots \int a_{2m} p^t(a^t) \sum_j \gamma_j^t a_{1j}^t da_{11}^t \cdots da_{2m}^t > \int a_{11} \cdots \int a_{2m} p^t(a^t) \sum_j \gamma_j^t a_{2j}^t da_{11}^t \cdots da_{2m}^t, \tag{13}
\]
then the expected utility for each student from not view the ranking is strictly increasing in the probability that university 1 is ranked first, \( r^t = (1, 2) \).

Theorem 3 indicates that a student who does not view the ranking prefers that the publication rank as number-one the university whose attribute scores she favors based on prior expectations. If the publication does so, then she gains the added prestige of choosing to attend the top-ranked university.

Because the optimal ranking methodology for students who view the ranking differs from the optimal one for students who do not, the socially-optimal ranking methodology is not student-optimal in general. In the next example, we characterize the socially-optimal ranking methodology and demonstrate that only in a special circumstance – when there is no prestige effect – is the socially-optimal ranking methodology also student-optimal.

5.1 Dynamic Example 1 Redux: A Comparison of the Publisher’s Optimal, the Student-Optimal, and the Socially-Optimal Methodologies

We characterize \( w_t^s \) for each of the periods as analyzed earlier in Dynamic Example 1. In our analysis we set the number of period-\( t \) ranking views, \( s_t \), at the equilibrium value. The table in Exhibit 4 lists the equilibrium and socially-optimal methodologies as well as the probability university 1 is ranked first by the socially-optimal ranking methodology, \( q^t((1, 2); w_t^s) \).

Following period 1, as pictured in Exhibit 4, the publisher-optimal and socially-optimal methodologies begin with the student-optimal ranking methodology but then diverge: the socially-optimal weight attached to attribute 1 increases, which in turn increases the probability in this ranking methodology that university 1 is ranked first. The main driver for this increase is the external effect of the ranking on the students who do not view it.

In this example, if a student does not view the ranking, then she chooses to attend university 1. With a prestige effect in place, the ranking methodology that maximizes the sum of the non-viewing students’ utilities therefore ranks the universities \((1, 2)\) with probability 1. As a result, in accounting for this external effect of the ranking on the students who do not view it, as the prestige effect grows over time, the socially-optimal ranking methodology increases the probability university 1 is ranked first by increasing \( w_t^s \) over time. Therefore, because the publisher decreases \( w_t^s \), and the universities are more likely to be ranked \((2,1)\), over time, the publisher’s optimal ranking methodology reduces social welfare, as expressed in (12), over time.

Note that the prestige effect drives the divergence of the socially-optimal and the publisher’s
optimal ranking methodologies. Without a prestige effect, as established in Theorem 1, the publisher optimally sets the student-optimal methodology. Without a prestige effect, the students who do not view the ranking are not concerned with the actual ranking. Therefore, without a prestige effect, the socially-optimal ranking methodology is the student-optimal methodology.

6 Discussion

6.1 Summary of Findings

We have demonstrated that whether or not the methodology that a publisher uses to rank universities aligns with student preferences depends on whether or not the publication creates prestige for universities by ranking them highly. If a university ranking has no prestige effect on student utility, then the publication’s optimal ranking methodology matches student preferences. But, if a university ranking in a publication with established views has a prestige effect, then the publication’s optimal ranking methodology diverges from student preferences. Furthermore, if a university ranking has a prestige effect, then a publication has an incentive to change its ranking methodology over time.

The incentive for a publication to change its university ranking methodology derives from the difference between the profit motive of the publications and the utility function of the students. If students were to place more weight on the informative role of rankings and less weight on the prestige of attending highly-ranked universities, then rankings publications would move their weights closer to those that are student-optimal. But it may actually be optimal for students to include a prestige effect term in their utility functions if, for example, the market for college graduates uses university rankings as signals of the quality of college graduates.

We have also shown that the ranking methodology that maximizes the sum of the publisher surplus, the surplus of students who view the ranking, and the external effect on the utility of the students who do not view the ranking differs from the one that matches student preferences and is likely to change over time. Furthermore, the socially-optimal and the publisher-optimal methodologies likely diverge, with the welfare-maximizing ranking moving toward stability of the rankings and the publisher’s optimal ranking methodology moving toward greater variation in the rankings.

6.2 Towards A Better Ranking System

If a publisher’s profit incentive were removed or if the prestige effect were eliminated, then the publisher would have an incentive to use a ranking methodology that matches student preferences.
Consider the publisher’s profit incentive; we do not see how a traditional private sector solution could work due to the incentive incompatibility issue that is central to our model. To achieve incentive compatibility, we need a stakeholder-neutral source that could gather the needed information and present it in an appropriate, peer-reviewed manner to the student-customers. Such a stakeholder-neutral source could emerge in one of two ways: organically or by construction. An organic solution might be a version of Wikipedia (“Wikiranks”?); a constructed answer might involve an organization like the American Association of Universities (AAU). In either case, the solution should involve a neutral organization providing a solution that addresses the heterogeneous needs of the student-customers, providing both attribute-level information and weighting suggestions for different student/customer segments. The solution could be pure attribute-and-aggregate or include some revealed-preference information. A revealed-preference solution incorporates non-evoked attributes through an intercept term.²⁰

We do not see deploying a revealed-preference approach alone as a viable solution (Avery et al., 2013). Because students can choose among only the universities that have admitted them, ranking universities based on student matriculation decisions is not simple and there may be competing reasonable methodologies. Private sector publishers will thus have an incentive to choose the methodology that maximizes the number of viewers, perpetuating the problem of an attribute-and-aggregate ranking system.

Now consider removing or reducing the “prestige effect.” Guidance counselors and other external influences can attempt to help students “choose the school that is best for you (them).” However, particularly since it may be rational to include a prestige effect in the student utility function (as a market-value signal to potential employers), it is hard to see how much progress could be made following this tack. So, it seems that the solution to the problem will likely rely on some sort of stakeholder-neutral solution, the form of which is beyond the scope of this article.²¹

### 6.3 Extensions, Limitations, and Further Research

Empirical analyses have identified the importance of product ranks, controlling for other factors such as product attributes, on consumer decision making (Simonsohn, 2011). In the case of colleges and universities, Luca and Smith (2009) and Griffith and Rask (2007) demonstrate that the actual ranks

²⁰ A revealed preference solution infers preference weights based on actual student choices and incorporates non-evoked attributes through an intercept term that captures residual “brand equity.” See Gul and Pesendorfer (2004).

²¹ We welcome readers’ creative input here.
in *USNWR*, controlling for college and university attributes, influence application and matriculation decisions. For hospitals, [Pope (2009)](#) finds that improvements in ranks of hospitals in *U.S. News: Best Hospitals* attracts more patients. [Sorensen (2007)](#) finds that positions in the *New York Times Book Review* affect book sales. [Dai et al. (2014)](#) construct an optimal mechanism to aggregate consumer ratings on Yelp.com. However, none of these studies have examined whether the influence of rankings on consumer choices is informative or persuasive.\(^{22}\) Our theoretical analysis underscores the need for establishing whether product and university rankings are (or should be) only informative or whether they should be expected to provide prestige as well.

One direction for further work involves competition, or the lack thereof, in our model. An open question is whether competition amongst publishers to rank universities would push the rankings closer to the student-optimal ranking methodology.

We have assumed that all students have the same relative preference for product attributes (i.e., all students have the same \(\gamma^t\) values). If students were to differ in the relative preferences for product attributes, and with competition among publication rankings, another intriguing question is whether different publishers would tune their methodologies to try to cater to the needs of different customer-segments. Whether such a result would be good or bad for the students is subject to Segal’s law.\(^{23}\) In either case, the issue is whether competition would encourage rankings that are student-optimal, at least within the targeted segment.

We hope we have demonstrated that the apparently arbitrary changes ranking publications make in their ranking methodologies for universities are not whimsical, but are driven by the logic of their business. Hence, the problem as we see it will not go away on its own. And we see two fundamental challenges: (a) (Further diagnosis): When can rankings of any sort be trusted to reflect the preferences of the targeted consumers? and (b) (Prescription): What can be done to make rankings more trustworthy? We hope that our work will both stimulate research in the field and encourage innovations in the practical world of ranking publications.

\(^{22}\) In consumer product markets, [Ackerberg (2001)](#) empirically distinguishes informative and prestige effects of advertising.

\(^{23}\) “A man with a watch knows what time it is. A man with two watches is never sure.” [Bloch, 2003](#) pg. 36.
References


A Appendix 1: Proofs

Proof of Lemma 1 Claim: For each \((\alpha^{t+1}, \beta^{t+1}) > 0\), \(V_{\text{Read}}^{t+1} - V_{\text{Not}}^{t+1}\) is strictly increasing in \(s^t\). Therefore, \(s^{t+1}\) is strictly increasing in \(s^t\).

Proof of Claim: We begin by proving the first sentence of the claim. The period-(\(t + 1\)) prestige effect, \(\beta^{t+1}g(s^t)R(r^{t+1})\), is increasing in \(s^t\). Furthermore, the increase in \(V_{\text{Read}}^{t+1}\) is greater than the increase in \(V_{\text{Not}}^{t+1}\). The reasoning is as follows. If the student does not view the ranking, then suppose she chooses university \(i\). Alternatively, if she views the ranking, she would choose either university \(i\) or one that is better-ranked. In the cases in which she chooses \(i\) after viewing the ranking, then an increase in \(s^t\) has the same effect on her ex-post utilities of viewing and of not viewing the ranking. However, if viewing the ranking leads to her choice of a better-ranked university, then an increase in \(s^t\) has a greater effect on her ex-post utility of viewing the ranking than of not viewing the ranking. We prove the second sentence of the claim. For a given \((\alpha^{t+1}, \beta^{t+1}) > 0\), an increase in \(V_{\text{Read}}^{t+1} - V_{\text{Not}}^{t+1}\) could shift the student from not viewing to viewing the ranking.

With the claim in place, we have that \(s^t(w^t) + s^{t+1}(w^{t+1}; s^t(w^t))\) is a strictly increasing transformation of \(s^t(w^t)\). Similarly, we have that \(s^{t+1}\) is strictly increasing \(s^t\). Therefore, \(s^{t+2}\) is strictly increasing in \(s^t\). Continuing in this manner, we have that \(\sum_{t \geq 1} s^t(\cdot)\) is a strictly increasing transformation of \(s^t(w^t)\). Therefore, if \(w^t\) maximizes \(s^t\), then \(w^t\) is the period-\(t\) element of the solution to
\[
\max_{w^t, t=1,2,\ldots} \sum_t s^t(w^t; s^{t-1}).
\]

Q.E.D.

Proof of Lemma 2 (i) If \(H^t_\alpha(0) < 1\) and \(H^t_\beta(0) = 1\), then
\[
V_{\text{Read}}^t(w^t; s^{t-1}) \equiv \alpha^t \sum_r q^t(r^t; w^t) \max_i \int_{a_{i1}} \cdots \int_{a_{inm}} p(a^t|r^t; w^t) \sum_j \gamma_j^t a_{ij}^t da_{i1}^t \cdots da_{im}^t \quad (14)
\]
and
\[
V_{\text{Not}}^t(w^t; s^{t-1}) \equiv \alpha^t \max_i \int_{a_{i1}} \cdots \int_{a_{inm}} p(a^t) \sum_j \gamma_j^t a_{ij}^t da_{i1}^t \cdots da_{im}^t. \quad (15)
\]
Next, from (14) and (15), a student views the ranking if and only if

\[ \alpha_t \geq \frac{1}{c} \left[ \sum_r q^t(r^t; w^t) \max_i \int_{a_{i1}} \cdots \int_{a_{in}} p(a^t \mid r^t; w^t) \sum_j \gamma_j \int_{a_{i1}} \cdots \int_{a_{in}} d^t a_{i1} \cdots d^t a_{in} - \max_i \int_{a_{i1}} \cdots \int_{a_{in}} p(a^t) \sum_j \gamma_j \int_{a_{i1}} \cdots \int_{a_{in}} d^t a_{i1} \cdots d^t a_{in} \right]. \]  

(16)

Choosing \( w_t \) to maximize \( s^t \) is equivalent to choosing \( w_t \) to minimize the r.h.s. of (16). Because \( V_{\text{Read}}^t \) is independent of \( w_t \), then minimizing the r.h.s. of (16) amounts to choosing \( w_t \) to maximize (14).

(ii) If \( H_\alpha^t(0) = 1 \), \( H_\beta^t(0) < 1 \), and a student views the ranking, then she chooses the top-ranked university, regardless of its identity. Her utility of viewing the ranking is:

\[ V_{\text{Read}}^t(w^t; s^{t-1}) \equiv \beta_t g(s^{t-1}) R(1) \]  

(17)

and a student’s expected utility of not reading a ranking is:

\[ V_{\text{Not} \text{Read}}^t(w^t; s^{t-1}) = \beta_t \max_i \int_{a_{i1}} \cdots \int_{a_{in}} p^t(a^t) \sum_r q^t(r^t \mid a^t; w^t) g(s^{t-1}) R(r^t) d^t a_{i1} \cdots d^t a_{in}. \]  

(18)

Next, from (17) and (18), a student views the ranking if and only if

\[ \beta_t \geq \frac{1}{g(s^{t-1}) R(1)} \max_i \int_{a_{i1}} \cdots \int_{a_{in}} p^t(a^t) \sum_r q^t(r^t \mid a^t; w^t) g(s^{t-1}) R(r^t) d^t a_{i1} \cdots d^t a_{in}. \]  

(19)

Choosing \( w_t \) to maximize \( s^t \) is equivalent to choosing \( w_t \) to minimize the r.h.s. of (19). Because \( V_{\text{Read}}^t \) is independent of \( w_t \), then minimizing the r.h.s. of (19) amounts to choosing \( w_t \) to minimize (18).

Q.E.D.

**Proof of Theorem 1.** (i) If \( H_\alpha^t(0) < 1 \) and \( H_\beta^t(0) = 1 \), then by Lemma 2 the publisher’s objective is to maximize (5). If the publisher ranks schools according to the student’s ordinal utility ranking (i.e., uses the student-optimal ranking methodology), then for each \( r^t \), the student matriculates at her preferred school. Therefore, the publisher’s ranking maximizes (5).

(ii) If \( H_\alpha^t(0) = 1 \) and \( H_\beta^t(0) < 1 \), then by Lemma 2 the publisher’s objective is to minimize (7).
If for some \( t' \),

\[
\arg\max_{\alpha} \int_{a_{11}} \ldots \int_{a_{nm}} p^t(a^t) \sum_{r^t} q^t(r^t|a^t; w^t) R(r^t) da_{11} \ldots da_{nm}
\]

\[
> \int_{a_{11}} \ldots \int_{a_{nm}} p^t(a^t) \sum_{r^t} q^t(r^t|a^t; w^t) R(r^t) da_{11} \ldots da_{nm},
\]

then the publication can change \( q_R(r^t|A)_{A\in A} \) to reduce (7). It does so by decreasing the probability that \( i \) is ranked first and increasing probability that \( i' \) is ranked first. Therefore, if the publication maximizes \( V_{Read} - V_{Not} \), then for each \( i, i' \),

\[
\int_{a_{11}} \ldots \int_{a_{nm}} p^t(a^t) \sum_{r^t} q^t(r^t|a^t; w^t) R(r^t) da_{11} \ldots da_{nm}
\]

\[
= \int_{a_{11}} \ldots \int_{a_{nm}} p^t(a^t) \sum_{r^t} q^t(r^t|a^t; w^t) R(r^t) da_{11} \ldots da_{nm}.
\]

This equality holds if \( q_R(r) = \frac{1}{n!} \). Because the student is admitted by all universities, this equality holds if and only if the probability university \( i, i \in N \) is ranked first is \( 1/n \). The publisher uses the uniform ranking methodology.

Q.E.D.

**Proof of Lemma 3.** By induction. Assume any two periods \( t' \) and \( t'' \) are ex ante identical in terms of student preferences and university attribute scores: \( \gamma^{t'} = \gamma^{t''} \), \( H^{t'}_{\alpha} = H^{t''}_{\alpha} \), \( H^{t'}_{\beta} = H^{t''}_{\beta} \), and \( p^t = p'^t \).

We have \( s^1 > s^0 = 0 \). Consider \( t \) and \( t - 1 \). Assume \( s^{t-1} > s^{t-2} \). If the the publisher in period \( t \) sets \( w^t = w^{t-1} \), then because the prestige effect is greater in period \( t \) than in period \( t - 1 \), we have that \( V^t_{View} - V^t_{Not} > V^{t-1}_{View} - V^{t-1}_{Not} \). Therefore, if \( w^t = w^{t-1} \), then \( s^t > s^{t-1} \). Because the publication has the option to set any \( w^t \) and not necessarily \( w^t = w^{t-1} \), we have that for any \( w^t, s^t > s^{t-1} \). Q.E.D.

**Proof of Theorem 2.** Consider the case in which \( \gamma_1 > w_1^{uniform} \), where \( w_1^{uniform} \) denotes the weight attached to attribute 1 in a uniform methodology. Derive the function \( \hat{\alpha}(w^t, s^{t-1}, \beta^t) \) from \( V^t_{View} - V^t_{Not} - c = 0 \). Write \( X^t \) as the expected utility from viewing the ranking less the expected utility from not viewing, considering only the utility of the attribute scores; and write \( Y^t \) as the expected utility from viewing the ranking less the expected utility from not viewing, considering only the prestige effect. Then,

\[
\hat{\alpha}(w^t, s^{t-1}, \beta^t) = \frac{c - \beta^t s^{t-1} Y^t(w^t_1)}{X^t(w^t_1)}.
\] (20)
For \( \alpha^t = 0 \), derive the function \( \hat{\beta}(w^t, s^{t-1}) \) also from \( V^t_{view} - V_{Nat} - c = 0 \). Using these functions and assuming \( h^t_\alpha = \frac{1}{\alpha} \) and \( h^t_\beta = \frac{1}{\beta} \), we can write the proportion of students who view the ranking in period \( t \) as:

\[
s^t(w^t; s^{t-1}) = \int_0^\infty \hat{\alpha}(w^t, s^{t-1}, \beta^t) \frac{1}{\alpha \beta^t} \, dw^t \, d\beta^t + \int_0^\infty \hat{\beta}(w^t, s^{t-1}) \frac{1}{\beta^t} \, d\beta^t,
\]

which equals

\[
s^t(w^t; s^{t-1}) = 1 - \frac{1}{\alpha \beta^t} \int_0^\infty \hat{\beta}(w^t, s^{t-1}) \alpha^t(w^t, s^{t-1}, \beta^t) \, d\beta^t.
\]

Differentiating (21) with respect to \( s^t \) using Leibniz rule, we have the first-order condition for the publication’s period-\( t \) maximization problem:

\[
\frac{\partial s^t(w^t; s^{t-1})}{\partial w^t_1} = -\frac{1}{\alpha \beta^t} \int_0^\infty \hat{\beta}(w^t, s^{t-1}) \frac{\partial \hat{\alpha}(w^t, s^{t-1}, \beta^t)}{\partial w^t_1} \, d\beta^t + \hat{\beta}(w^t, s^{t-1}) \frac{\partial \hat{\beta}(w^t, s^{t-1})}{\partial w^t_1} = 0.
\]

Next, we take the total differential of (22) with respect to \( w^t_1 \) and \( s^{t-1} \) and set it equal to zero (so that the first-order condition (22) continues to be satisfied):

\[
\frac{\partial^2 s^t(w^t; s^{t-1})}{\partial w^t_1^2} dw^t_1 + \frac{\partial^2 s^t(w^t; s^{t-1})}{\partial w^t_1 \partial s^{t-1}} ds^{t-1} = 0.
\]

Rearranging (23) and using the property that the second-order condition requires that \( \frac{\partial^2 s^t(w^t; s^{t-1})}{\partial w^t_1^2} < 0 \), we have that

\[
\text{sign} \left| \frac{dw^t_1}{ds^{t-1}} \right| = \text{sign} \left| \frac{\partial^2 s^t(w^t; s^{t-1})}{\partial w^t_1 \partial s^{t-1}} \right|.
\]

In determining \( \frac{\partial^2 s^t(w^t; s^{t-1})}{\partial w^t_1 \partial s^{t-1}} \), we use Leibniz rule and the property that \( \hat{\alpha}(w^t, s^{t-1}, \beta^t) = 0 \) and \( \frac{\partial \hat{\alpha}(w^t, s^{t-1}, \beta^t)}{\partial s^{t-1}} = 0 \) when \( \hat{\beta} \) is evaluated at \( \beta^t \). We have

\[
\frac{\partial^2 s^t(w^t; s^{t-1})}{\partial w^t_1 \partial s^{t-1}} = -\frac{1}{\alpha \beta^t} \int_0^\infty \hat{\beta}(w^t, s^{t-1}) \frac{\partial \hat{\alpha}(w^t, s^{t-1}, \beta^t)}{\partial w^t_1} \, d\beta^t.
\]

In evaluating the sign of (25), we have that

\[
\frac{\partial^2 \hat{\alpha}(w^t, s^{t-1}, \beta^t)}{\partial w^t_1 \partial s^{t-1}} = \frac{\beta^t Y^t \frac{\partial X^t}{\partial w^t_1}}{(X^t)^2} > 0
\]

because \( \frac{\partial X^t}{\partial w^t_1} > 0 \) for \( w^t_1 \in [w^t_{uniform}, \gamma^t_1] \). Therefore,

\[
\frac{\partial^2 \hat{\alpha}(w^t, s^{t-1}, \beta^t)}{\partial w^t_1 \partial s^{t-1}} < 0.
\]
Finally, from (24) and (25), we have that
\[
\frac{dw^t_1}{ds^{t-1}} < 0. \tag{27}
\]
For the case in which \( \gamma_1 < w^\text{uniform}_1 \), from Lemma 3 (i.e., \( s \leq s^{t-1} \)) and (27), we have that for any \( t \) and \( t - 1 \), \( w^t_1 < w^{t-1}_1 \).

The proof for the case in which \( \gamma_1 < w^\text{uniform}_1 \), the proof to establish that for any \( t \) and \( t - 1 \) that \( w^t_1 > w^{t-1}_1 \) follows the same path as the proof for the case in which \( \gamma_1 > w^\text{uniform}_1 \).

Q.E.D.

Proof of Theorem 3. We assume that for each student,
\[
\int_{a_{11}^t} \cdots \int_{a_{2m}^t} p^t(a^t) \sum_j \gamma_j^t a_{1j}^t da_{11}^t \cdots da_{2m}^t > \int_{a_{11}^t} \cdots \int_{a_{2m}^t} p^t(a^t) \sum_j \gamma_j^t a_{2j}^t da_{11}^t \cdots da_{2m}^t. \tag{28}
\]
Therefore, if \( \beta^t = 0 \), a student who does not view the ranking would choose to attend university 1. Now, add the prestige effect to the student’s utility function (\( \beta^t > 0 \)). If the student attends university 1 and university 2 is ranked first, then she would be strictly better off if she attends university 1 and this university were ranked first. If the student attends university 2 because this university is ranked first, then she would be strictly better off if she were to switch to attending university 1 and this university were ranked first. Therefore, for each student who experiences a prestige effect and does not view the ranking, her expected utility is maximized if university 1 is ranked first and she attends this top-ranked university.

Q.E.D.

B Appendix 2: Dynamic Examples

Setup for Examples

Each of our examples has two universities and each university has two attributes.

We begin our description of the examples by writing the students’ utility functions. A period-\( t \) student’s utility of attending school 1 is
\[
U^t_1 = \begin{cases} 
\alpha^t (\gamma^t_1 a_{11}^t + a_{12}^t) + \beta^t s^{t-1} R(1) & \text{if } r^t = (1, 2); \\
\alpha^t (\gamma^t_1 a_{11}^t + a_{12}^t) + \beta^t s^{t-1} R(2) & \text{if } r^t = (2, 1),
\end{cases} \tag{29}
\]
where $\gamma_2^t$ is normalized to 1 and $g(s^{t-1}) = s^{t-1}$. The students’ strength of preference for attribute 1, $\gamma_1^t$, is such that $q^t((1, 2), \gamma_1^t) > 1/2$. This assumption means that if the publication sets $w_1^t = \gamma_1^t$, then university 1 is ranked first with a probability greater than 1/2.

We normalize university 2’s attribute scores to zero. After doing so, a student’s period-$t$ utility of attending school 2 is:

$$U_2^t = \begin{cases} \beta^t s^{t-1} R(1) & \text{if } r^t = (1, 2); \\ \beta^t s^{t-1} R(2) & \text{if } r^t = (2, 1). \end{cases}$$ (30)

University 1’s attribute score $a_{11}^t$ is distributed on $[0, 1]$ according to a beta distribution:

$$p_{11}(a_{11}^t) = \frac{a_{11}^t \phi_1^{-1} (1 - a_{11}^t)^{\psi_1^{-1}}}{\int_0^1 y^{\phi_1-1} (1 - y)^{\psi_1-1} dy};$$ (31)

and university 1’s attribute score $a_{12}^t$ is distributed on $[-1, 0]$ also according to a beta distribution:

$$p_{12}(a_{12}^t) = -\frac{a_{12}^t \phi_2^{-1} (1 + a_{12}^t)^{-\psi_2^{-1}}}{\int_0^1 y^{\phi_2+1} (1 - y)^{\psi_2-1} dy}. \hspace{1cm} (32)$$

In specifying $a_{11}^t > 0 = a_{21}^t$, $a_{12}^t < 0 = a_{22}^t$, university 1 neither dominates nor is dominated by university 2. With these possible values in place, the first attribute is university 1’s core competency and the second is university 2’s core competency.

The students’ values of the university attributes, $\alpha^t$, are distributed on $[0, \overline{\alpha}^t]$ according to the beta distribution:

$$h_{\alpha}(\alpha^t) = \frac{(\alpha^t)^{\tau-1} (1 - \alpha^t)^{\theta-1}}{\int_0^{\overline{\alpha}^t} (y^{\tau-1} (1 - y)^{\theta-1}) dy};$$ (33)

and the students’ values of the publication’s ranking, $\beta^t$, are distributed on $[0, \overline{\beta}^t]$ according to the beta distribution:

$$h_{\beta}(\beta^t) = \frac{(\beta^t)^{v-1} (1 - \beta^t)^{\varphi-1}}{\int_0^{\overline{\beta}^t} (y^{v-1} (1 - y)^{\varphi-1}) dy}. \hspace{1cm} (34)$$

We normalize the publication’s weight attached to attribute 2 at $w_2^t = 1$. Therefore, the publication’s period-$t$ ranking methodology is the weight it attaches to attribute 1, $w_1^t$. Note that publication’s optimal period-$t$ weight $w_1^t$ lies between the weight of the uniform ranking methodology and the weight of the student-optimal ranking methodology.

**Dynamic Example 1**

In this two-school, two-attribute case the values of attending the top- and second-ranked universities
are $R(1) = 1.1$ and $R(2) = 1.0$. The student places twice the weight on attribute 1 scores as she does on attribute 2 scores, $\gamma_1^t = 2$. We skew the distributions of the students’ preferences, $\alpha^t$ and $\beta^t$, so that in expectation the students place greater weight on the prestige of the ranks than on the university attributes: the student values of the university attributes $\alpha^t$ are distributed $[0, \alpha^t] = [0, 1000]$ according to a nonsymmetric beta distribution $h^t_\alpha(\alpha^t)$, as specified in \text{(33)} with $\tau = 1$ and $\theta = 2$. The students’ values of the publication’s ranking $\beta^t$ are distributed $[0, \beta^t] = [0, 1000]$ according to a nonsymmetric beta distribution, as specified in \text{(34)} with $\upsilon = 2$ and $\varphi = 1$. The cost to each student, whether in terms of time or money, of viewing the publication is $c = 20$.

University 1’s attribute score $a_{11}^t$ is symmetrically distributed on $[0, 1]$, as specified in \text{(31)} with $\phi_1 = \psi_1 = 1$; and its attribute score $a_{12}^t$ is symmetrically distributed on $[-1, 0]$, as specified in \text{(32)} with $\phi_2 = \psi_2 = 1$.

**Dynamic Example 2**

We retain the numerical values specified in dynamic example 1, with the exception of setting the initial value of $\gamma_1^t$ at 2.1 and then decrease the value beginning in period 6 to 2.0.
<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Publication’s ranking methodology</td>
<td>Weights attached to attribute scores</td>
</tr>
<tr>
<td>Student-optimal ranking methodology</td>
<td>Ideal ranking methodology for students who view ranking</td>
</tr>
<tr>
<td>Uniform ranking methodology</td>
<td>Each possible ranking is equally likely</td>
</tr>
<tr>
<td>Welfare-maximizing ranking methodology</td>
<td>Maximizes sum of publisher’s profit and utilities of students who view and do not view ranking</td>
</tr>
<tr>
<td>Utility of attending university</td>
<td>Determined with university attribute scores and rank in place</td>
</tr>
<tr>
<td>Expected utility of attending university</td>
<td>Determined prior to learning attribute scores and rank</td>
</tr>
<tr>
<td>Expected utility of viewing ranking</td>
<td>Determined prior to viewing ranking and accounts for matriculation decision made after viewing</td>
</tr>
<tr>
<td>Expected utility of not viewing ranking</td>
<td>Accounts for matriculation decision made, and if prestige effect is present, ranking affects it</td>
</tr>
<tr>
<td>Prior probabilistic belief of attribute scores</td>
<td>Prior to viewing ranking, students hold these beliefs</td>
</tr>
<tr>
<td>Posterior probabilistic belief of attribute scores</td>
<td>After viewing ranking, students hold these beliefs</td>
</tr>
<tr>
<td>Probabilistic belief about particular ranking</td>
<td>Students hold these beliefs, and they are conditional on ranking methodology</td>
</tr>
</tbody>
</table>
Table 2: Summary of Basic Notation

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description of Notation (all in period $t$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^t_{ij}$</td>
<td>University $i$’s value of attribute $j$</td>
</tr>
<tr>
<td>$w^t_j$</td>
<td>Weight the publication attaches to attribute $j$</td>
</tr>
<tr>
<td>$r^t_i, r^t_{ri}$</td>
<td>Publication’s ranking and university $i$’s position in the ranking</td>
</tr>
<tr>
<td>$R(r^t_i)$</td>
<td>Student value of publication’s rank of university $i$</td>
</tr>
<tr>
<td>$s^t$</td>
<td>Number of students who view the ranking</td>
</tr>
<tr>
<td>$g(s^{t-1})$</td>
<td>Student value of number of students who view the ranking</td>
</tr>
<tr>
<td>$\alpha^t, \beta^t, \gamma^t_j$</td>
<td>Weights in student utility function</td>
</tr>
<tr>
<td>$p^t_{ij}(a^t_{ij})$</td>
<td>Probability density function for value of university $i$’s attribute $j$</td>
</tr>
<tr>
<td>$h^t_\alpha(\alpha^t), H^t_\alpha(\alpha^t), h^t_\beta(\beta^t), H^t_\beta(\beta^t)$</td>
<td>Student population density, distribution function of $\alpha^t$, $\beta^t$ respectively</td>
</tr>
<tr>
<td>$q^t(r^t; w^t)$</td>
<td>Student probabilistic belief ranking is $r^t$, given $w^t$</td>
</tr>
<tr>
<td>$U^t_i$</td>
<td>Student utility of attending university $i$</td>
</tr>
<tr>
<td>$E[U^t_{i, \text{View}}], E[U^t_{i, \text{Not}}]$</td>
<td>Student expected utility of attending university $i$ if view, not view</td>
</tr>
<tr>
<td>$V^t_{\text{Read}}, V^t_{\text{Not}}$</td>
<td>Student expected utility of viewing, not viewing the ranking</td>
</tr>
</tbody>
</table>
Exhibit 1: Example 1. In this table and figure, the publisher’s weight attached to attribute 1 begins at the student’s weight, $w_1^{1*} = \gamma_1 = 2.0$, and then as the number of views increases over time, the attribute weight moves away from 2.0 over eight periods to eventually converge on $w_1^{8*} = w_1^{9*} = 1.64$.

<table>
<thead>
<tr>
<th>Period</th>
<th>Publisher Weight $w_t^{1*}$</th>
<th>Student Views $s_t^1$</th>
<th>Probability ranking is $(1,2)$ $q_t^{(1,2); w_t^{1*}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2.00</td>
<td>0.332</td>
<td>0.825</td>
</tr>
<tr>
<td>2</td>
<td>1.85</td>
<td>0.482</td>
<td>0.800</td>
</tr>
<tr>
<td>3</td>
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<td>0.564</td>
<td>0.785</td>
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<tr>
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<td>0.613</td>
<td>0.774</td>
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<tr>
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<td>0.767</td>
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<td>1.66</td>
<td>0.661</td>
<td>0.762</td>
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</tr>
<tr>
<td>9</td>
<td>1.64</td>
<td>0.686</td>
<td>0.755</td>
</tr>
</tbody>
</table>
Exhibit 2: Three methodologies for Example 1: the student-optimal, $w_1^t = \gamma_1^t = 2$; a uniform, $w_1^t = 1$; and an intermediate, $w_1^t \in (1, 2)$. In the shaded area, the intermediate ranking methodology ranks school 2 first, but the students would prefer school 1 to be ranked first.
Exhibit 3: Example 2. In this table and figure, a jump in student attribute-1 weight in period 6 causes a lagged, several-period response by the publication in changing its ranking methodology.

<table>
<thead>
<tr>
<th>Period</th>
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<th>Publisher Weight $w_1^t$</th>
<th>Student Views $s^t$</th>
<th>Probability ranking is (1,2) $q^t((1,2);w_1^t)$</th>
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Exhibit 4: Comparison of Efficient and Publisher’s Optimal Methodology. The efficient weight attached to attribute 1 begins at the student-optimal weight, $w_1^{1*} = \gamma_1 = 2.0$, and then as the number of views grows, this efficient attribute weight converges to the $w_1^{1*} = ... = w_1^{9*} = 2.17$. 

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<th>Efficient Weight $w_1^{1*}$</th>
<th>Student Views $s^t$</th>
<th>Probability ranking is (1,2) $q^t((1,2),w_1^{1*})$</th>
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