The University Rankings Game:
Modeling the Competition among Universities for Ranking

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ABSTRACT

With university rankings gaining both in popularity and influence, university administrators develop strategies to improve their rankings. To better understand this competition for ranking, we develop an adjacent category logit model to address the localized nature of ranking competition and include lagged rank as an independent variable to account for stickiness of ranking. Calibrating our model with data from USNews from 1999-2006 shows persistence in ranking and identifies important interactions among university attributes and lagged rank. The model provides (lagged) rank-specific elasticities of ranks with respect to changes in university characteristics, thereby offering insight about the effect of a university’s strategy on its rank.

Key words: Adjacent category logit model, university ranking, USNews
The [university] system has become “marketized” in the sense that its participants need increasingly to think of themselves in business terms. A whole industry of “enrollment management” consultants has arisen to handle what is ordinarily known as “admissions” and was once quaintly called “crafting a class.”


1. INTRODUCTION

Environmental changes, particularly the recent marked increase in public availability of information, are resulting in the US higher education system becoming “marketized” (Geiger 2004). Universities are driven to act like firms in competitive marketplaces, seeking effective competitive strategies. Competition among universities to enroll students, hire faculty, raise funds, and to improve their rankings published in magazines such as the U.S. News and World Report’s America’s Best Colleges (from hereon USNews) has significantly increased in recent years. University administrators increasingly rely on rankings as marketing tools, since rising university costs and decreasing governmental funding has increased the intensity of competition among universities (Hossler 2000; Hunter 1995). According to Machung (1998), universities use rankings to attract students, to bring in alumni donations, to recruit faculty and administrators, and to attract potential donors - all key performance metrics. Our primary objective here is to develop a model of competition among universities for ranking that provides insights into the nature of this competition.

Aside from higher education, rankings are prevalent in other industrial sectors including product-specific ranked lists from Consumer Reports, the Environmental Protection Agency’s “Nifty Fifty List” of top chemical polluters, the Hot 100 Billboard songs, the BCS college football rankings, and others. Academics have paid attention to these ranking issues by developing models that account for the specific features of the ranking context. For example, Bradlow and Fader (2001) use the Billboard Hot 100 ranking data to calibrate a generalized gamma latent worth function and develop a Bayesian lifetime model for songs. Other research has focused on methods for developing rankings (e.g., ranking of statistics journals – Theoharakis and Skordia 2003), and on
critiquing existing ranking systems (e.g., BCS ranking of NCAA Division 1-A college football – Frey 2005; Mease 2003; Stern 2004).

In USNews, university ranks are partly based on two broad categories of university attributes – institutional resources and reputation – which normally change slowly (for USNews procedure see http://www.usnews.com/usnews/edu/college/rankings/about/04rank_brief.php). For example, the 1999-2006 lists of USNews top 50 universities include 47 that appear annually, with Harvard, Princeton, Stanford, and Yale all in the top 5 each year. Thus, we expect persistence to exist in university rankings, i.e., lagged rank should contain information about current rank. Rankings are based on attributes such as selectivity in admissions but do not explicitly include persistence, our focus here. And when a university gains in rank, another must lose; if ranks are sticky, the universities most likely to lose are those with similar ranks. Therefore, competition for ranks tends to be localized, where gain or loss of ranks occurs within a few ranks at a time. For the 1999-2006 USNews data, the average absolute change in rank in a one-year period was 1.53, suggesting that competition is localized among similarly ranked universities. Given this localized competition, we develop an adjacent category logit model that addresses interactions among university attributes and persistence in ranking (Goodman 1983; Simon 1974).

The results from our adjacent category logit model demonstrate persistence in university ranking and localized competition. The persistence of ranking results show that lagged rank is a key driver of current rank and that lagged rank interacts in a strategically important manner with university attributes such as academic reputation, financial resources, and faculty resources. For example, we find that improving the academic reputation subrank results in a greater change in overall rank for lower ranked universities (Rank 40) than for higher ranked universities (Rank 10). Hence, our analysis of persistence shows that the competition to improve ranks among lower ranked universities is different from the competition to do so among higher ranked universities. Our results
also support the rank-localized nature of competition among universities, where the competition is primarily among similarly ranked universities. For example, our results show that the top ranked university has a .965 probability of finishing in the top 5 the next year.

2. UNIVERSITY RANKING HISTORY

University rankings first appeared in the 1870s with the objective of informing higher education scholars and professionals, and government officials (Stuart 1995). Rankings gained mass appeal in 1983, almost one century after they were first introduced, when USNews, using a survey of university presidents, published its first rankings of undergraduate academic quality. In 1987, USNews adopted its current multidimensional methodology, aggregating more objective attributes along with assessments by academic leaders of their peer institutions.

When USNews first introduced its university ranking issue, the publication ranked the top-25 national universities and top-25 national colleges. In 1998, USNews expanded its rankings of the national universities to the top-50 universities. In the 2004 ranking, USNews created three categories – national doctoral universities, regional master’s universities, and colleges. The latest version – the 2006 issue – ranks 120 national doctoral universities and 104 national liberal arts colleges.

As the USNews rankings are the oldest and most popular, they have been the subject of several academic studies (Avery, Fairbanks, and Zeckhauser 2003; Ehrenberg and Monks 1999). Ehrenberg and Monks, 1999, find that a drop in rank leads a university to accept a greater percentage of applicants, a smaller percentage of its acceptance pool matriculates, a lower quality entering class, and a need to offer more financial incentives to attract applicants. Avery et al. (2003) analyze the practice of early admissions and find that universities admissions decisions are favorable to early admission candidates, consistent with a policy that would improve their ranks.
It appears that while university administrators sometimes criticize published rankings, they clearly recognize that these rankings as public performance scorecards. For example, Hobart and William Smith Colleges fired a senior vice president in 2000 after she failed to submit fresh data to USNews, resulting in a major drop in the College’s rank (Graham and Thompson 2001). In fact, Richard Beeman, Dean of the College of Arts and Sciences at the University of Pennsylvania, in a letter to the New York Times (Sept 17, 2002) commented “… I breathed a sigh of relief when my university continued to appear in the [USNews] top 10.”

3. MODELING FRAMEWORK AND METHODOLOGY

3.1. Modeling Framework

Based on the issues discussed above, we specify our model by recognizing that (1) university ranks are sticky and (2) any change in ranks will happen in incremental steps, i.e., a few ranks at a time. Thus, we adopt an adjacent category logit model framework (e.g., Goodman 1983; Simon 1974). The adjacent category logit model accommodates explanatory variables and provides probabilities for change in rank given the change in the value of explanatory variables. The model is:

$$\log(\frac{\pi_r(X_{ur}, Y_{u(t-1)})}{\pi_{r+1}(X_{ur}, Y_{u(t-1)})}) = \alpha_r + Y_{u(t-1)}\beta + X_{ur}\gamma + Y_{u(t-1)}X_{ur}\delta$$

(1)

where, r, u, and t index ranks, universities, and time (year) respectively, $\alpha_r$, $\beta$, $\gamma$, and $\delta$, are parameters to be estimated and capture the rank-specific intercept term, effect of lagged rank ($Y_{u(t-1)}$), effect of subranks ($X_{ur}$), and interaction between the lagged rank and subranks respectively.

We define the probability $\pi_r(X_{ur}, Y_{u(t-1)})$ as:

$$\pi_r(X_{ur}, Y_{u(t-1)}) = P(Y_{ur} = r | X_{ur}, Y_{u(t-1)}) = \frac{\exp(\alpha_r + Y_{u(t-1)}\beta_r + X_{ur}\gamma_r + Y_{u(t-1)}X_{ur}\delta_r)}{1 + \sum_{r=1}^{R}\exp(\alpha_r + Y_{u(t-1)}\beta_r + X_{ur}\gamma_r + Y_{u(t-1)}X_{ur}\delta_r)}$$

(2)

where, $\beta_r = (R - r)\beta$, $\gamma_r = (R - r)\gamma$, and $\delta_r = (R - r)\delta$ (e.g., Agresti 2002).
We assume the likelihood function is a simple product of the probabilities in Equation 2 when
the lagged rank is explanatory, implying a Markov-type structure (cf., Agresti 2002, Section 11.5), as
well as cross-sectional independence of ranks, after conditioning on prior rank. While these
assumptions are violated to a degree (for example, time series structure more complex than
Markovian are likely, and ranking data are structurally dependent), we performed several checks that
showed that the violations are not severe. (Our data are available upon request for those wishing to
explore other models or analytic structures).

3.2. Data Sources and Variable Operationalization

USNews publishes the overall ranking of the top universities along with subrankings on key
aggregated attributes: academic prestige rank, graduation and retention rank, selectivity rank, faculty
resources rank, financial resources rank, and alumni giving rank (we refer to these six aggregated
attributes as subranks). USNews uses pre-defined weights to combine the scores on the attributes in
each of these six categories into an overall score. From the university management’s perspective, to
gain in the USNews rankings, a university must invest in improving one or more of the six subranks.
Even if a university takes no actions to improve one or more of these subranks, its subranks still
may change. That is, a gain in overall rank for a university implies some other university is losing
rank and the strategies (intentional or unintentional) that universities follow are reflected in changes
in their subranks. For example, one university may decide to improve its faculty while another may
focus on its graduation and retention subrank.

To determine ranks, USNews begins by calculating the scores of the attributes in each of the six
subcategories, and then builds a final score by calculating a weighted sum of the attribute scores. It
then ranks the universities by the final scores. In determining the overall ranks, if two schools
receive the same integer scores (e.g., 79.49 and 79.01), then USNews reports a tie between the
universities. In determining the subranks (for faculty resources, for example) USNews scores each
university, calculating the weighted sum of attribute scores in the faculty resources subcategory, and then ranks the universities in the faculty resources subcategory according to these subcategory scores. In our analysis, we examine the relationship between the subranks of the six subcategories (for which we create subrankings) and the overall ranks.

We use eight years of data from USNews – 1999 to 2006 – of the top-50 universities. Although USNews has published data since 1983, the methodology has changed periodically, with the last major change occurring in 1999 when USNews moved from a four-point to five-point scale in its peer assessment survey. We consider only the 47 universities that were in the top-50 rankings for the eight-year period (1999-2006) in the analysis, giving us 329 data-points. (See Table 1 for descriptive statistics and bivariate correlation coefficients).

[Insert Table 1 about here]

3.3. Alternate Model Specifications

We test the viability of the proposed model (M_{HY}) against alternate model specifications to ensure that (1) the model is not over-specified such that a constrained version outperforms the hypothesized model, and (2) the model is not subject to omitted variable bias. We first specified two simpler configurations that are nested in the hypothesized model: (1) (M_{LR}) which models lag of rank as the sole explanatory variable and (2) (M_{MAIN}), which excludes interaction terms between lag of rank and the subranks and thus only includes main effects. Second, we estimated two models that incorporate time-specific (M_{TS}) and university-specific (M_{US}) fixed effects and we also estimated a model that accounts for unobserved heterogeneity by examining the possibility of multiple regimes (i.e., latent segments; M_{FM}). We use the minimum CAIC value (Consistent Akaike Information Criterion) as a model selection criterion.
4. RESULTS

4.1. Model Selection

The hypothesized model \( M_{\text{HYP}} \): \( LL = -454.6, NP = 62, \text{CAIC} = 1209.8 \) (where \( LL \) = log likelihood value, \( NP \) = number of parameters, \( \text{CAIC} \) = Consistent Akaike Information Criterion) outperforms both the model with lag of rank as the only explanatory variable \( M_{\text{LRK}} \): \( LL = -701.2, NP = 50, \text{CAIC} = 1644.4 \) and the model main effects model that does not include the interactions among the subranks and the lag of rank \( M_{\text{MAIN}} \): \( LL = -599.1, NP = 56, \text{CAIC} = 1469.9 \). The hypothesized model also outperforms the time-specific fixed effects model \( M_{\text{TS}} \): \( LL = -447.8, NP = 68, \text{CAIC} = 1225.4 \), the university-specific fixed effects model \( M_{\text{US}} \): \( LL = -362.0, NP = 108, \text{CAIC} = 1247.8 \)), and the model with two latent regimes \( M_{\text{FM}} \): \( LL = -369.8, NP = 125, \text{CAIC} = 1400.0 \).

The rank-specific intercept terms for the hypothesized model are well behaved in that similar ranks have values for the intercept term closer to each other than for dissimilar ranks; see Figure 1.

4.2. Influence of Explanatory Variables

We present the results for \( M_{\text{HYP}} \) in Table 2. Since we focus on interactions between lagged rank and subranks, the main effects parameters do not have substantive interpretations. Hence, we illustrate the use of the model graphically.

The scatter plot in Figure 2 reveals a marked persistence in university ranking; that persistence remains even when one models the subranks that USNews uses to arrive at its ranking. To further investigate the persistence of ranks, we display results from \( M_{\text{LRK}} \) and the probabilities of change in rank with previous rank being 5, 15, 25, 35, and 45 in Table 3. These probabilities show the localized nature of rank competition among the universities, supporting our persistence hypothesis. For example, the university ranked 15th has probabilities of .048, .057, .114, and .225 to be ranked
11th, 12th, 13th or 14th respectively the following year. In fact the probability of this university being ranked within four of its current rank 15 is .903. An anomaly evident from Table 3 is illustrated by the 15th ranked university, which has a probability of 0.119 to be ranked 18th compared to a probability of 0.063 to be ranked 16th the next year. Such anomalies are an artifact of the USNews approach to dealing with tied ranks. For example, if two universities are both given a rank of 2, the next university is given a rank of 4, skipping the rank of 3.

Persistence also interacts in meaningful ways with the subranks, providing insights into strategies that universities might use to improve their rankings. Upward sloping curves for the plot of these interactions (Figure 3) suggest that as subranks improve so does the rank. For the university ranked 10, financial resources and graduation and retention are the top two subranks (top plot in Figure 3), whereas for a university ranked 40, academic reputation and graduation and retention are the two critical subranks (bottom plot in Figure 3). In contrast, alumni giving and selectivity subranks appear to be the least important subranks for a university ranked 10 (top plot in Figure 3), while for a university ranked 40 alumni giving and financial resources are the least important subranks (bottom plot in Figure 3). Thus, irrespective of a university’s rank, it should focus on graduation and retention and should not expect much return by increasing emphasis on alumni giving more than its competition. A highly ranked university gets more leverage from growing financial resources while lower ranked universities get more leverage from improvements in academic reputation.

For the top universities we report the probability of a university to be ranked in the top 9 given it is ranked in top 5 the previous year (Table 4). For the period from 2000-2006, eight universities were ranked in the top 5. Of these Harvard, Princeton, Stanford, and Yale made it to the top 5 in
each of the seven years; Massachusetts Institute of Technology and University of Pennsylvania were
there for six of the seven years; Cal Tech was in the top 5 for five years; and Duke was in the top 5
for four years. For this elite group, the probability of losing rank is fairly low (as seen by the
probability of being ranked from 6 to 9). The top ranked university has a 0.375 probability of
retaining the top position, a 0.207 probability of coming in second and a 0.965 probability of a top 5
rank in the next year. The second ranked university has a probability of 0.291 to improve its rank, a
probability of 0.185 to maintain its rank, and a probability of 0.937 of ending up in the top 5 the
next year. As one would expect, the probability of finishing in the top 5 steadily declines as we
move from rank 3 to 5, going from 0.890 to 0.708.

[Insert Table 4 about here]

5. FURTHER RESEARCH DIRECTIONS AND LIMITATIONS

University rankings are gaining in importance for university administrators as well as academics
concerned with understanding the nature of the academic environment. It is important and timely
for researchers to examine the competitive dynamics that stem from competition for ranking among
universities. It is our hope that this research will spawn more interest in understanding the
competitive dynamics concerning university rankings and that other researchers will explore this
domain, using or expanding upon the data we have collected.

Other model structures should address some of the limitations of our analysis here. For
example, the significant interaction terms between lagged rank and subranks may be an artifact of
ceiling effects that may disappear with an appropriate latent modeling strategy. The model we chose
is highly parameterized and more parsimonious models can surely be developed that might provide
different perspective. And it would be useful to investigate time series and cross-section
dependence issues in the context of a more comprehensive model. Clearly, other research avenues
are also possible and should be pursued.
Table 1. Descriptive statistics and bivariate correlation coefficients: Statistics are presented for N = 329 (U = 47, T = 7) data points. All correlation coefficients are statistically significant (p < .01). Multicollinearity is a possible concern due to the high correlations.

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>RK</th>
<th>LRK</th>
<th>ACAD</th>
<th>GRAD</th>
<th>FAC</th>
<th>SEL</th>
<th>FIN</th>
<th>ALUM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank (RK)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lagged Rank (LRK)</td>
<td>.99</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Academic Reputation Rank (ACAD)</td>
<td>.84</td>
<td>.84</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Graduation and Retention Rank (GRAD)</td>
<td>.79</td>
<td>.80</td>
<td>.58</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Faculty Resources Rank (FAC)</td>
<td>.67</td>
<td>.65</td>
<td>.32</td>
<td>.44</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Selectivity Rank (SEL)</td>
<td>.82</td>
<td>.79</td>
<td>.69</td>
<td>.63</td>
<td>.50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financial Resources Rank (FIN)</td>
<td>.64</td>
<td>.64</td>
<td>.48</td>
<td>.37</td>
<td>.52</td>
<td>.38</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alumni Giving Rank (ALUM)</td>
<td>.59</td>
<td>.58</td>
<td>.29</td>
<td>.54</td>
<td>.51</td>
<td>.44</td>
<td>.32</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>23.49</td>
<td>23.43</td>
<td>23.80</td>
<td>25.26</td>
<td>35.40</td>
<td>26.08</td>
<td>31.78</td>
<td>47.95</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>13.61</td>
<td>13.61</td>
<td>14.80</td>
<td>16.64</td>
<td>34.65</td>
<td>19.26</td>
<td>27.59</td>
<td>52.02</td>
</tr>
</tbody>
</table>
Table 2. Results from the adjacent category logit model ($M_{\text{HYP}}$).

<table>
<thead>
<tr>
<th>Variable Category</th>
<th>Variable Name</th>
<th>Coefficient (Standard Error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank Persistence</td>
<td>Lag of Rank (LR)</td>
<td>.0807** (.0200)</td>
</tr>
<tr>
<td></td>
<td>Academic Reputation (ACAD)</td>
<td>-.0320** (.0130)</td>
</tr>
<tr>
<td></td>
<td>Graduation and Retention Rank (GRAD)</td>
<td>.0061 (.0105)</td>
</tr>
<tr>
<td></td>
<td>Faculty Resources (FAC)</td>
<td>.0136 (.0084)</td>
</tr>
<tr>
<td></td>
<td>Selectivity (SEL)</td>
<td>-.0090 (.0128)</td>
</tr>
<tr>
<td></td>
<td>Financial Resources (FIN)</td>
<td>.0367** (.0093)</td>
</tr>
<tr>
<td></td>
<td>Alumni Giving (ALUM)</td>
<td>.0022 (.0055)</td>
</tr>
<tr>
<td>Subranks</td>
<td>LR *ACAD</td>
<td>.0050** (.0006)</td>
</tr>
<tr>
<td></td>
<td>LR *GRAD</td>
<td>.0018** (.0003)</td>
</tr>
<tr>
<td></td>
<td>LR *FAC</td>
<td>.0004* (.0002)</td>
</tr>
<tr>
<td></td>
<td>LR *SEL</td>
<td>.0012** (.0004)</td>
</tr>
<tr>
<td></td>
<td>LR *FIN</td>
<td>-.0006* (.0003)</td>
</tr>
<tr>
<td></td>
<td>LR *ALUM</td>
<td>.0002 (.0002)</td>
</tr>
</tbody>
</table>

* p < .05
** p < .01
Table 3. Probability of rank persistence and change with lagged rank \(a_5\) to current rank \(ab\). The table gives the probability that a university will obtain a current rank of \(ab\) when its previous year's rank was \(a_5\). Thus, if in year \(t-1\) the university is ranked 15 (\(a = 1\)) then the university has a probability of .063 of being ranked 16 in year \(t\) (\(b = 6\)). Specifically, we go down the second column (as \(a = 1\)) and sixth row (as \(b = 6\)). Similarly, a university ranked 25\(^{th}\) (\(a = 2\)) in the year \(t-1\) has a .020 probability of being ranked 24\(^{th}\) (\(b = 4\)) in the year \(t\).

<table>
<thead>
<tr>
<th>Probability of Current Rank (ab)</th>
<th>Lagged Rank (a_5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a = 0)</td>
</tr>
<tr>
<td>(b = 1)</td>
<td>.086</td>
</tr>
<tr>
<td>(b = 2)</td>
<td>.083</td>
</tr>
<tr>
<td>(b = 3)</td>
<td>.061</td>
</tr>
<tr>
<td>(b = 4)</td>
<td>.220</td>
</tr>
<tr>
<td>(b = 5)</td>
<td>.257</td>
</tr>
<tr>
<td>(b = 6)</td>
<td>.057</td>
</tr>
<tr>
<td>(b = 7)</td>
<td>.080</td>
</tr>
<tr>
<td>(b = 8)</td>
<td>.035</td>
</tr>
<tr>
<td>(b = 9)</td>
<td>.079</td>
</tr>
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</table>
Table 4. Probability of rank persistence and change among Top 5 universities.

<table>
<thead>
<tr>
<th>Current Year Rank</th>
<th>Actual Ranks</th>
<th></th>
<th></th>
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<tr>
<td></td>
<td>Previous Year Rank</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>.375</td>
<td>.291</td>
<td>.212</td>
<td>.142</td>
<td>.086</td>
</tr>
<tr>
<td>2</td>
<td>.207</td>
<td>.185</td>
<td>.154</td>
<td>.119</td>
<td>.083</td>
</tr>
<tr>
<td>3</td>
<td>.087</td>
<td>.089</td>
<td>.086</td>
<td>.076</td>
<td>.061</td>
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<tr>
<td>5</td>
<td>.119</td>
<td>.161</td>
<td>.205</td>
<td>.241</td>
<td>.257</td>
</tr>
<tr>
<td>6</td>
<td>.015</td>
<td>.024</td>
<td>.034</td>
<td>.047</td>
<td>.057</td>
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<td>7</td>
<td>.012</td>
<td>.022</td>
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<tr>
<td>9</td>
<td>.004</td>
<td>.009</td>
<td>.021</td>
<td>.042</td>
<td>.079</td>
</tr>
<tr>
<td>Cumulative Probability of Top 5</td>
<td>.965</td>
<td>.937</td>
<td>.890</td>
<td>.815</td>
<td>.708</td>
</tr>
<tr>
<td>Cumulative Probability of Top 9</td>
<td>.999</td>
<td>.997</td>
<td>.993</td>
<td>.982</td>
<td>.958</td>
</tr>
</tbody>
</table>

**NOTES:** See note on Table 3 for explanation.
Figure 1. Distribution of rank-specific intercept terms, which shows that similar ranks have values closer to each other than for dissimilar ranks.
Figure 2. Scatter plot for Rank and Lagged Rank, which shows persistence of rank overall. In the plot we use .5 standard deviation jitters so that duplicates are not hidden.
Figure 3. Effect of lag rank and subranks on current rank. The top figure shows the plot for the effect of subranks on rank when lagged rank is 10 and the bottom figure depicts the same effect when the lagged rank is 40. These plots show that for better-ranked universities (lagged rank 10), financial resources and graduation and retention subranks are the most important, whereas for relatively lower ranked universities (lagged rank 40), academic reputation and graduation and retention subranks are the most critical. For any subrank (S) and lag of rank (L), we calculate the effect of rank as 
\[ \alpha S + \beta S L \]
where \(\alpha\) is the main effect of subrank and \(\beta\) is the interaction between the subrank and lag of rank. Thus, \(\alpha\) and \(\beta\) are estimated values and we plug in the values of S and L.
REFERENCES


